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REGULATION OF LAKE ONTARIO[a]

Franklin F. Snyder,[1] M. ASCE and Robert H. Clark[2]
(Proc. Paper 1660)

SYNOPSIS

After a description of the physical characteristics of the Great Lakes and e hydrology of Lake Ontario, the regulation studies of the Great Lakes dur- g the past half century are outlined briefly. The latest international udies for regulation of Lake Ontario and problems of special interest en- untered therein are described.

INTRODUCTION

The regulatory structures and the increased capacities of the St. Lawrence ver in the International Rapids Section, which are an integral part of the St. wrence Seaway and Power projects, provide a means of exercising as com- ete a control of the level of Lake Ontario and the flow of the St. Lawrence ver as the conflicting interests involved will permit. Under date of 25 June 52, the Governments of Canada and the United States referred to the Inter- tional Joint Commission the problem of determining, having regard to all her interests, whether measures could be taken to regulate the level of ke Ontario for the benefit of property owners on the shores of the lake in e United States and Canada, so as to reduce the extremes of stage which had en experienced. Among other things, the Commission was directed to termine whether, in its judgment, action could be taken by either or both vernments to bring about a more beneficial range of stage having regard

te: Discussion open until November 1, 1958. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. Paper 1660 is part of the copyrighted Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers, Vol. 84, No. HY 3, June, 1958.

Presented at a meeting of the ASCE, Buffalo, N. Y., June, 1957.

Chief, Reservoir Regulation Section, Office of the Chief of Engrs., U.S. Dept. of the Army, Washington, D. C.

Chief Hydr. Engr., Water Resources Branch, Dept. of Northern Affairs and National Resources, Ottawa, Canada.

to the proposed plan for improvement of navigation and power in the International Rapids Section of the St. Lawrence River, and the proposed method of regulation of the levels of Lake Ontario which was an essential feature of that plan.

The proposed plans for improvement of the river and for the regulation of Lake Ontario were set forth in the Applications of the Governments of Canada and the United States to the International Joint Commission for the development of power in the International Rapids Section. These applications were approved by the Commission four months after receiving the Reference on lake levels. The Order of Approval, dated 29 October 1952, referred specifically to Method No. 5, developed in 1940 by the Special Projects Branch, Department of Transport, Canada, as a regulation plan for Lake Ontario, and all works connected with the planning of the power and navigation scheme, including the channel excavations and backwater computations, had been based on the levels and flows produced by this method of regulation. Because revisions had been made to certain of the basic flow data used in Method No. 5 and since that method was not designed to meet the terms of the Reference of 25 June 1952, further regulation studies were necessary.

To undertake the studies required under the Reference, the International Joint Commission appointed the International Lake Ontario Board of Engineers in April 1953. This Board is composed of one representative from each Government; Mr. Gail A. Hathaway, formerly Special Assistant to the Chief of Engineers, Department of the Army, Washington, D. C., now Engineering Consultant to the International Bank for Reconstruction and Development, as the United States member, and Mr. T. M. Patterson, Director, Water Resources Branch, Department of Northern Affairs and National Resources, as the Canadian member. The Board was instructed to undertake through the appropriate agencies in Canada and in the United States, the necessary investigation and studies, and to advise the Commission on all technical engineering matters which it must consider in making a report or reports to the two Governments under the Reference of 25 June 1952.

The purpose of this paper is to summarize the studies of the International Lake Ontario Board of Engineers and present briefly the factors which must be taken into consideration in the development of a plan of regulation for Lake Ontario.

Description of the Great Lakes

The Great Lakes basin, constituting the major part of the St. Lawrence River System,has a drainage area, both land and water, of approximately 295,000 square miles, at the head of the St. Lawrence River at the easterly end of Lake Ontario. Of this area about 41 per cent lies in Canada and the remainder in the United States. The International Boundary passes through all of the Great Lakes and their connecting channels, excepting Lake Michigan which is wholly within the United States, and through the St. Lawrence River to a point near Cornwall, Ontario. Downstream of this point the St. Lawrence River is wholly within Canada.

The size of the four upper Great Lakes has a very marked effect upon the magnitude of the fluctuations of Lake Ontario, and an understanding of the physiography and hydrography of the St. Lawrence-Great Lakes basin is a prerequisite to a study of the regulation of any one of the Great Lakes. A

eneral location map of the basin is shown on Figure 1. For the purpose of his paper the principal characteristics of the Great Lakes are set out in 'able 1.

Lake Ontario, the lowest in the Great Lakes chain is also the smallest ith a water surface area of about 7,500 square miles. The local drainage asin area of Lake Ontario, exclusive of the area of the lake itself, is about 7,300 square miles. During the past 95 years (1860-1954) the net total suply to Lake Ontario, i.e. all of the water reaching the lake minus the evaporaion from its surface, has averaged about 241,000 cubic feet per second, and he monthly supplies have ranged from a low of about 135,000 cubic feet per econd for October 1934, to a high of about 419,000 cubic feet per second for pril 1870. Based on a 95-year average, about 85 per cent of the net total upply to Lake Ontario consists of inflow from the other Great Lakes through he Niagara River, and about 15 per cent consists of supply from the Lake ntario basin.

A representation of the average variation in the net supply is given in 'igure 2. An analysis of the various components of the net supply is also epicted. While it is evident that the outflow from Lake Erie makes up the reater part of the total net supply to Lake Ontario, it is also evident that he fluctuations in the supply are mainly the result of the variations in the unoff from the local drainage area and the precipitation-evaporation balance n the lake surface. It is of interest to note also that the evaporation from he lake surface exceeds the precipitation in about seven months of the year. n a monthly basis the supply from the Lake Ontario basin has varied from a ow in certain winter months of considerably less than the quantity of water vaporated from the lake's surface during the month, to a high in certain arly spring months of over 50 per cent of the net total supply for the month.

Diversions of water into and out of the Great Lakes basin over the past eriod of record have changed the natural supply to the lakes. Diversion into ake Superior from the Albany River basin, which is tributary to Hudson Bay, hrough the Long Lake and Ogoki projects in Canada commenced in 1939 and 943 respectively. Since 1945 the sum of these diversions has averaged bout 5,000 cubic feet per second. Diversions from Lake Michigan at Chicago nto the Mississippi River basin commenced about 1848 and averaged 500 ubic feet per second until 1900 and thereafter increased progressively until maximum annual average of about 10,000 cubic feet per second was reached 1 1928; it then decreased progressively from 1929 through 1938. Since 1938, 1 accordance with the United States Supreme Court Decree of April 21, 1930, he diversion has been maintained at an annual average of 1,500 cubic feet per econd, exclusive of domestic pumpage, which has averaged about 1,600 cubic eet per second, resulting in a total diversion of 3,100 cubic feet per second ut of the Great Lakes basin. Based on these averages, the net change in ater supply to Lake Ontario is an increase of 1,900 cubic feet per second. 1 the regulation studies, the computed natural supplies to Lake Ontario, that s, the supplies which would have obtained through the period of record since 860 without any diversion into or out of the Great Lakes basin, have been inreased by a constant amount of 1,900 cubic feet per second.

The average level of Lake Ontario at Oswego, New York, for the period 860 to 1954 was elevation 246.04 feet, U.S.L.S. 1935 datum. The monthly ean level of the lake has varied from a low of 242.68 feet for November 934, to a high of 249.29 feet for June 1952, making a total range in stage of .61 feet.

TABLE 1. - DATA PERTAINING TO THE GREAT LAKES

Lake	Drainage Area: Water Surface Area Square Miles	Drainage Area: Land Area Square Miles	Average Elevation m.s.l.	Range of Monthly Mean Stage	Outlet River	Mean Outflow c.f.s.
Superior*	31,800	48,200	602.3	4.09	St. Mary R.	75,000
Michigan	22,400	45,500	580.6	6.24	St. Clair R.	
Huron	23,000	49,600	580.6	6.24	Lake St. Clair and Detroit R.	189,000
Erie	9,900	29,400**	572.4	5.41	Niagara R.	205,000
Ontario	7,500	27,300	246.0	6.61	St. Lawrence R.	241,000
Totals	94,600	200,000				

* Controlled under the supervision of the International Lake Superior Board of Control.

** Includes Lake St. Clair and its local drainage area.

Because of the natural regulatory influence of the Great Lakes the dis-harge of the St. Lawrence River at the outlet of Lake Ontario is remarkably teady. At Iroquois, Ontario, the monthly mean discharge over the past 95 ears has averaged 241,000 cubic feet per second and has ranged from a naximum of 323,000 cubic feet per second in May 1870, to a minimum of 54,000 cubic feet per second in February 1936. Immediately below Lake t. Louis, in which a portion of the Ottawa River discharge joins that of the t. Lawrence, the monthly mean discharge has averaged 278,000 cubic feet er second and has ranged from a maximum of 487,000 cfs in May 1876 to a ninimum of 170,000 cubic feet per second in February 1936.

Description of the St. Lawrence River

From the outlet of Lake Ontario at Kingston, Ontario, to Father Point, Quebec, which marks its transition into the Gulf of St. Lawrence, the St. Lawrence River falls 246 feet in a distance of 533 miles. The major portion of this fall, some 226 feet, occurs between Lake Ontario and Montreal, 183 niles downstream from the lake. Most of the remaining 20 feet of fall occurs n the 160 miles between Montreal and Quebec City.

The development in the International Rapids Section includes a dam in the icinity of Iroquois Point to control the levels of Lake Ontario, a dam in the Long Sault Rapids between Barnhart Island and the United States shore and wo powerhouses, one on either side of the International Boundary at the foot of Barnhart Island, with a total installation rated at 2.4 million horsepower. The dam, dikes and relocations for the power pool below the Iroquois control dam are designed to provide for full Lake Ontario level but initially the Order of Approval requires the pool to be operated at a maximum elevation of 238.0 feet at the powerhouse. Excavations in the river are being designed o provide a controlling depth of 27 feet for navigation and to provide satis-actory velocities for navigation and power.

A few miles below the Barnhart powerhouses, the river widens into Lake t. Francis with a surface area of about 90 square miles. With the exception f about three miles of shoreline, this lake lies in Canada. The level of the ake was under partial control from 1932 to 1943, but since that time it has een under full control by the Beauharnois power plant and a series of dams t its natural outlet near Coteau Landing. The method of operation requires hat Lake St. Francis be maintained at levels which would have occurred nder natural conditions, although in practice the range of stage has been re-uced during periods of extreme high or low flows. The local inflow to the ake comes from a drainage area of approximately 4,400 square miles.

From the outlet of Lake St. Francis the river narrows and falls a distance f about 83 feet in 15.5 miles before discharging into Lake St. Louis. The edars power plant, built in 1914, utilizes about 35 feet of this drop and has n installed capacity of 197,000 horsepower. The Beauharnois power plant, hich is on the shore of Lake St. Louis, develops the head between Lakes St. rancis and St. Louis. The rating of the present installation is 1,425,000 orsepower, but construction now under way will increase the rated capacity o 2,160,000 horsepower. The forebay of the plant is connected to Lake St. rancis by a combined power and navigation canal, about 16 miles long and ,000 feet wide.

Lake St. Louis has a surface area of 56 square miles and, in addition to

the inflow from the St. Lawrence River, receives a portion of the runoff of the Ottawa River through the Vaudreuil and St. Anne outlets of the Lake of Two Mountains as well as the runoff from approximately 1,300 square miles of local drainage. The percentage of the total Ottawa River flow discharged into Lake St. Louis varies from about 50 per cent, under high flow conditions, to about 20 per cent under low flow conditions; the remainder flows through the Rivière des Mille Iles and Rivière des Prairies to join the St. Lawrence River below Montreal Island. The drainage area of the Ottawa River is 57,000 square miles and the maximum and minimum daily inflows to Lake St. Louis from the Ottawa River have been 190,000 cubic feet per second and 3,000 cubic feet per second respectively, while the monthly means have ranged from 160,000 cubic feet per second to 4,000 cubic feet per second. A one-foot change in the level of Lake St. Louis at mean stage is equivalent to a change in discharge of about 30,000 cubic feet per second.

From the outlet of Lake St. Louis to Montreal Harbour, a distance of about 13 miles, the fall in the river is about 47 feet, 33 feet of which occurs through the Lachine Narrows and Rapids before the river widens to form Laprairie Basin, a shallow lake of about 29 square miles. Various schemes for developing the power in the Lachine Section have been proposed, with the final plan still not formulated. The plan which is receiving present consideration envisages one powerhouse to develop the 35 feet of head available at the Lachine Rapids upstream, with the possibility of another powerhouse downstream utilizing the remaining 15 feet. The development of the Lachine Section of the river for power will require considerable excavation through the Lachine Narrows at the outlet of Lake St. Louis to reduce velocities sufficiently to allow an ice cover to form. Both plans contemplate the control of Lake of Two Mountains on the Ottawa River.

The water levels in Montreal Harbour during the navigation season vary with the flow of the St. Lawrence and Ottawa Rivers and are influenced by the inflow from the tributaries entering the St. Lawrence River below Montreal. The minimum recorded level at Lock 1, Montreal Harbour, was elevation 16.6 feet on 20 October 1934, and the maximum recorded during the open water season was elevation 34.4 feet on 15 May 1876. The maximum open water levels occur during the spring runoff when Lake Ontario outflows are increasing and the Ottawa River discharge reaches a maximum.

An accurate determination of the relation between river discharge past Montreal and the open water river stage is complicated by the fact that these stages are modified by the varying discharge of the tributaries entering the river below Montreal, including the discharge of the Ottawa River through the Rivière des Mille Iles and the Rivière des Prairies and are affected by long term tidal fluctuations, as well as continual changes in outlet due to dredging in the St. Lawrence Ship Channel. A change in flow of 22,000 cubic feet per second changes the level in Montreal Harbour at low stages by about one foot. In winter the increased slope of the river, due to ice retardation, results in a rise of 11 to 22 feet in the water surface in Montreal Harbour, and ice jams, occurring during the break-up season, have raised the water elevation 32 feet above low water level.

History of Lake Ontario Regulation

For over half a century the problem of regulating the Great Lakes has been under active consideration and has become increasingly complicated as

more and more technical and social implications of the problem have come to light.

The series of dry years which culminated in 1895 provoked considerable clamour from navigation interests on the Great Lakes to have the lakes regulated to a substantially high, uniform level. This situation resulted in the appointment by the two Federal Governments of separate and joint commissions and boards to study the problem. The most important of these groups was the International Waterways Commission, whose deliberations extended from 1905 to 1913. The duty of this Commission, among other things, was to investigate and report on the maintenance and regulation of suitable levels on the Great Lakes, and also upon the effect upon the shores of these waters and the structures thereon. In its report on the regulation of Lake Erie, which was published in 1910, the Commission touched briefly on the regulation of all of the other Great Lakes. The primary objective of the regulation of Lake Erie was for the improvement to navigation, and some of the interesting features of this report are summarized:

a) The regulation was to be achieved by predicting the supply to the lake one month in advance by adding or subtracting an average constant to the known supply for the previous month and regulating the outflow in such a manner as to obtain a certain predetermined level on the lake at the end of that month. Discharges were limited to the channel capacity at any level.

b) Riparian interests on the lakes were considered from the point of view of maximum stages only, while those downstream did not appear to have been given any consideration. The effect of regulation of any of the lakes on downstream navigation was considered carefully.

c) It was assumed that regulation could be accomplished by changing the setting of the control gates once a month. The reason given for this was "The irregularity and occasional violence of its (Lake Erie's) oscillations make it necessary to take the average of considerable observations - usually a month - to find the true level. The gates can, therefore, be set not often than once a month and then only approximately."

The conslusions set out in the report with respect to the regulation of all the Great Lakes were quite similar to those of H. M. Chittenden and J. A. Seddon.* The latter, despite the limitations imposed by the data available at that time, outlined very clearly the forces at work in the natural regulation of the Great Lakes and concluded that the reduction in the fluctuations of the levels of Lakes Superior and Michigan-Huron would materially increase fluctuations on their connecting rivers and the downstream lakes; also that a regulation plan to reduce the fluctuations in the lower lakes by regulating the upper ones would not be too effective because these fluctuations result mainly from the variation in the local supply. In other words, the natural regulation now afforded by the lakes is very effective.

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*Reservoir System of the Great Lakes of the St. Lawrence Basin; Its relation to the Problem of Improving the Navigation of these Bodies of Water and of Their Connecting Chambers; With a Mathematical Analysis of the Influences of Reservoirs upon Stream Flow.
Transactions A.S.C.E. Vol. XL, 1898.

Beginning around 1913 the possibility of power development on the St. Lawrence River began to assume considerable importance. In 1920 studies on the proposed St. Lawrence Deep Waterway revealed that it was economically desirable to combine the navigation and power developments in the International Section of the river. Although interests of power and navigation in any regulation plan do not necessarily conflict, the requirements for power are more complicated. Navigation interests are concerned with the maintenance of water levels, whereas the power interests are concerned with both water levels and discharges, as well as successful operation under ice conditions.

Between 1921 and 1927 many reports and articles were published dealing with the regulation of the Great Lakes. Some of the most interesting of these are:

1) "Report on the St. Lawrence River from Montreal to Lake Ontario, Made to the International Joint Commission," by Colonel W. P. Wooten, Corps of Engineers, United States Army, and Mr. W. A. Bowden, Chief Engineer, Department of Railways and Canals, Canada. This report, completed in 1921, included separate studies by the Canadian Government Engineer and United States Government Engineer on investigations and conclusions as to the proper regulation of Lake Ontario. These regulation plans were developed for a combined power and navigation project on the St. Lawrence River and the main objectives were, therefore, to increase the minimum lake levels, to increase the minimum Lake Ontario outflows, and to make possible the formation of an ice cover during the winter months in the restricted reaches in the International Rapids Section by reducing the flow and thus the velocity. Consideration was given to the maintenance of low water levels in Montreal Harbour.

2) Statement and Engineering Report by the Hydro-Electric Power Commission of Ontario to the International Joint Commission Respecting the Proposal to Develop the St. Lawrence River, 1921.

3) Report of the Joint Board of Engineers on the St. Lawrence Waterway Project, 16 November 1926. Although this Board was interested primarily in power and navigation, its instructions also required a report on the question, "To what extent may water levels in the St. Lawrence River at and below Montreal, as well as the river and lake levels generally be affected by the execution of the project?" As a result of a thorough study of the river and lake levels the requirements of the riparian interest, both upstream and downstream of the International Rapids Section, were clarified and set out in the criteria governing the Board's regulation plans. This concept introduced additional complications in the regulation studies as the requirements for power and navigation conflict with the requirements of the riparian owners. The Joint Board of Engineers studied also the overall regulation of the Great Lakes and reached conclusions similar to those of Chittenden and Seddon.

Some of the more interesting aspects of its regulation studies are:

a) The method proposed for the overall regulation of the Great Lakes involved a forecast of the supply for the following month as a function of

the known supply of the previous month, and to regulate the outflow to achieve a desirable level at the end of the month consistent with the proposed channel capacities. The "B" approach which is described in a subsequent section is similar to this method.

b) The method proposed for the regulation of Lake Ontario only was very similar to that proposed by the Canadian Government Engineer in 1921. The regulated discharge was a function of the water level at the beginning of the month with a correction factor based on Lake Huron, rather than the Lake Erie levels used in the 1921 report. The "A" approach which is described in a subsequent section is similar to this method.

c) The requirements of the navigation, power and riparian interests, both upstream and downstream of the International Rapids Section, were set out as a guide to test the adequacy of regulation plans.

The report of the Joint Board of Engineers seemed to crystalize opinion on the regulation of Lake Ontario, and from 1925 to 1952 the concepts remained the same. In 1940 Mr. Guy A. Lindsay, Director, Special Projects Branch, Department of Transport, Canada, produced Method of Regulation No. 5, an adjustment of the method proposed by the Joint Board, to provide for the dry years from 1930 to 1936. Method No. 5 was later extended to include the high water of 1952.

The high lake levels of 1952 resulted in a demand for the regulation of the levels of Lake Ontario which would result in benefits to the lakeshore riparian interests. The studies of the International Lake Ontario Board of Engineers included this factor as an additional requirement of regulation.

Co-ordination of Basic Data

Because of the large amount of basic data on water levels and discharge measurements available in the files of governmental agencies of Canada and the United States, and the variations in the results from analyses of these data, it became evident at the outset of the study that it would be necessary for these federal agencies to reach agreement on all pertinent basic hydrologic and hydraulic data in Lake Ontario and in the International Rapids Section. To accomplish this, there was established a Co-ordinating Committee on Great Lakes Basic Hydraulic and Hydrologic Data composed of three representatives of federal agencies in each country having responsibilities in their respective countries for such basic data.

On Lake Ontario and through the International Rapids Section of the St. Lawrence River, water level records have been maintained since 1860. With these records and the discharge measurements made by both countries in the International Rapids Section since 1911, monthly outflows for the period 1860 to 1917, and daily outflows for the period 1917 to 1954 were computed for Lake Ontario. It was in 1917 that recording gauges were substituted for many of the staff gauges along the river.

To analyze the effects of regulation of Lake Ontario on the water levels and flows in the Montreal area, it was necessary to have a record of the past outflows from Lake St. Louis. These were computed by governmental agencies of Canada and were based on the discharge records of the St. Lawrence River for the International Rapids Section and the Ottawa River

at Grenville, Quebec. The inflows from about 7,000 square miles of intervening drainage area as well as the flow out of Lake of Two Mountains through the Rivière des Prairies and Rivière des Mille Iles were based on available discharge records and on appropriate correlations. Water level records in the wholly Canadian Section of the St. Lawrence River, including Lake St. Louis and Montreal Harbour also date back to 1860.

The computed outflows from both Lake Ontario and Lake St. Louis were adjusted to the present net diversion of 1,900 cubic feet per second into the Great Lakes system. Thus, it was assumed that the present diversions had been in effect throughout the period of record. The supplies to Lake Ontario used in the regulation studies were determined from the adjusted mean monthly or semi-monthly outflow and the change in storage during the corresponding period. The change in storage was determined using the available mean daily hydrographs of lake levels at Oswego, Port Dalhousie, Toronto and Kingston, and graphically eliminating the effects of winds and other factors.

To provide a uniform base against which to compare the regulated levels of Lake Ontario, the recorded levels were adjusted for the successive man-made changes which had affected the levels over the past. Thus, the comparison is made to those levels which would have occurred if the March 1955 configuration of the control channels at the Galop Rapids and the present diversions had been in existence since 1860, and with crustal movement occurring as it did in nature. The recorded adjusted levels are therefore not those which would be obtained if the supplies of the past were routed through the lake with the March 1955 configuration and elevation of the control channels in the Galop Rapids. Differential crustal movement has increased the elevation of the lake outlet with respect to the land around the lake. The significance of this phenomena may be illustrated by comparing the elevations on lake Ontario required to discharge the same flow near the beginning and end of the period of record; for example, in October 1870 a discharge of 272,000 cfs would have occurred with the lake at elevation 246.98 feet under present diversion and outlet conditions, whereas in July 1954 the same discharge with the same diversion and outlet conditions would have occurred with the lake at elevation 247.44 feet.

The recorded water levels of Lake St. Louis were also adjusted to take into account the effect of the past changes to its outlet and the changes in rates of diversion into and out of the Great Lakes basin. No adjustments were made to take into account the effect of storage facilities in the Ottawa River basin on the recorded water levels of Lake St. Louis.

Requirements of Various Interests

Navigation, riparian and power interests concerned with the regulation of the levels and outflows of Lake Ontario have divergent opinions as to the range of stage to be established for any method of regulation. The desires of these interests and the factors to be taken into consideration in connection with them are discussed hereunder.

Navigation

The navigation interests are concerned with maintaining a relatively high minimum level on Lake Ontario, which allows greater drafts for vessels, as

well as with maintaining high minimum flows in the St. Lawrence River to assure satisfactory downstream levels. This latter aspect is of particular importance in the Montreal area where it is essential to ensure that the minimum flow is not reduced, or the frequency of occurrence of low flow materially increased, due to the regulation of the outflows of Lake Ontario. This problem is complicated by the possibility of coincidence of low regulated outflow from Lake Ontario with low flows from the Ottawa River.

Navigation interests are concerned also to a lesser degree with maximum water levels insofar as they govern elevations of their structures as well as with the frequency and duration of high flows insofar as they affect river currents and velocities.

Riparian Interests Upstream

Shore property on Lake Ontario is subject to damage by flooding and by wave action. Damage results from flooding during the high lake stages but by far the greater damage is caused by wave action during storms, particularly during periods of high lake levels. This results in erosion of the shore and damage to or destruction of structures, buildings, and valuable arable land.

The shore of Lake Ontario varies from sparsely populated agricultural and undeveloped areas to the more developed sections in and adjacent to localities such as Rochester, Great Sodus Bay, and Oswego, New York, and Toronto, Hamilton and Cobourg, Ontario. In many communities extensive construction has taken place on low, unstable barrier beaches, as well as lakeward from the back shore at elevations below that considered reasonably safe from flooding and wave action. The construction consists generally of summer homes, permanent residences, and various types of commercial establishments including docks and marine service facilities.

There are many factors other than mean lake level which have a direct effect on damage to shore property from the waters of Lake Ontario. These factors include winds and waves, temporary fluctuation in levels, the time of year, the preceding succession of levels which have obtained and the geological formation of the area. The incidence of storms and temporary rises in the lake surface coupled with strong onshore winds are major factors in the damage caused to shore property by the waters of Lake Ontario. Fortunately the storms are generally less frequent and less severe during the summer season when the lake is at its highest seasonal stage.

Superimposed upon the mean lake level are daily and even hourly fluctuations resulting from unbalance or tilting of the lake's surface. These are induced primarily by winds and differential barometric pressures. A maximum rise of about three feet has been recorded at the east end of the lake.

It is difficult to establish a critical mean lake level from the standpoint of damage from wave action. Surveys and inspections during the past four years indicate that damage from wave action is inconsequential when the mean lake level is at or below the long term average elevation of about 246.0 feet. Damage from wave action increases progressively as the mean lake level rises. Such damage is intensified by temporary rises of the water surface above the mean lake level which are often accompanied by severe storms. As evidenced by the results of a survey in the spring of 1952, damage may be very severe from wave action when the mean lake stage ranges between elevation 247.5 feet and the maximum stages. Temporary rises of from one to one

and one-half feet, superimposed upon the mean lake level at those elevations, raise the water surface such that protective beaches are drowned out and many protective structures are of inadequate height to prevent overtopping. Waves wash over at many localities along the shore and attack the back shore and shore structures causing severe erosion and property damage.

It is estimated that severe damage from flooding begins when the water surface is between elevations 248.0 and 249.0, and becomes progressively more severe at higher stages. The water surface elevation may be the same as the mean lake level or it may be higher because of a temporary rise.

Any beneficial plan for regulation of the outflows from Lake Ontario would produce levels different from those which would obtain without regulation. Changes in lake levels and their duration will have an effect on the damage to shore property from flooding and wave action. Shore property interests desire a reduction of high lake levels. However, in developing a plan of regulation which will benefit shore property on Lake Ontario, consideration must also be given to the duration of intermediate and high levels, as well as the maximum level that is reached under the plan.

Some Lake Ontario riparian interests propose that the level of the lake be maintained at about its mean elevation of 246.0. In order to accomplish this it would be necessary to keep the outflows from the lake equal to the supplies. With supplies adjusted to present diversion conditions the discharges would vary from 145,000 cubic feet per second to 421,000 cubic feet per second. Discharge of the larger supplies would be entirely impracticable.

Riparian Interests Downstream

These include all interests, bordering on Lake St. Francis, Lake St. Louis, and the other reaches of the river below the International Rapids section, which are affected by the flows and levels of the St. Lawrence River. Lake St. Francis, with an area of 90 square miles and a storage capacity of 1,000 cubic feet per second per month per foot of depth and Lake St. Louis, with an area of 56 square miles and a storage capacity of 600 cubic feet per second per month per foot of depth, may be compared to Lake Ontario with an area of 7,540 square miles and a storage capacity of 80,000 cubic feet per second per month per foot of depth. With so little storage in Lakes St. Francis and St. Louis, they can have only minor effects in moderating the discharges of Lake Ontario.

The major problems with which downstream riparian interests are concerned are outlined hereunder:

a) High Open Water Levels on Lake St. Louis and Downstream: These usually occur during April, May or June coincident with floods on the Ottawa River. In order not to aggravate this problem, the regulated outflow from Lake Ontario at these times should not be greater than the corresponding outflows without regulation of Lake Ontario.

b) Low Levels on Lake St. Louis and Downstream: Lake St. Louis, Laprairie Basin and other sections of the St. Lawrence River downstream are used extensively for boating and other recreational uses, for domestic water supply and for sewage disposal purposes. Any lowering of the low levels would be detrimental to these uses, and since a large portion of these areas is very shallow, any lowering of the low levels during the period mid-May to mid-September would be particularly detrimental to the boating and recreational uses.

c) Ice Conditions in the Montreal Area: The St. Lawrence River, particularly in the Montreal area, is subject to ice jams with the most severe occurring below and in Montreal Harbour, and in Laprairie Basin. In the early stage of winter the southerly and northerly parts of Laprairie Basin cover with ice, but a central channel near Nuns Island remains open until the ice pack which starts in Lake St. Peter, about 50 miles below Montreal, builds upstream past Montreal, under Victoria Bridge, and into the Basin. While the pack below Montreal is building upstream, the water level at Montreal gradually rises until the head of the pack passes that point. After that, it falls slightly and remains at a constant level until the break-up period brings down large quantities of frazil and slush ice and raises the water level again. The early winter rise in Montreal Harbour varies from 11 to 22 feet. With continued cold weather the level at the head of the Laprairie Basin continues to rise slowly as more and more ice is brought to it from above, and sometimes the Lachine Rapids are drowned out for very short periods. In general, the highest level recorded at the upper end of Laprairie Basin is coincident with the last period of cold weather in February or March. Usually at that time the water level is about 11 feet above ordinary summer levels. Under these conditions the surface slope in the ice-gorged section between Lachine Rapids and Montreal is about 1.6 feet per mile.

In April, warm rains and sun weaken the surface ice which holds the hanging dams in place and a large quantity of surface ice, frazil and slush moves from its wide berth in Laprairie Basin to the narrow restricted river below Victoria Bridge. This movement constricts the channel further and is often accompanied by large rises in water level. Rises of 20 and 16 feet above summer stage for similar discharges are frequently found in Montreal Harbour and the Laprairie Basin respectively. It is believed that the operation of ice breakers below Montreal in recent years has somewhat reduced the flood hazard due to ice jam. The ice breakers, by working upstream from Lake St. Peter toward Montreal and clearing the channel as they proceed, attempt to provide an open channel upstream to the harbour.

Power Interests in the International Rapids Section

These power interests desire generally high lake Ontario stages to provide greater heads combined with a range of stage adequate to improve the distribution of lake outflows for greater firm power capacity.

It would be advantageous for the power entities responsible for the channel enlargements above the powerhouses in the International Rapids section to have the range of stage at high elevations. High minimum and maximum levels would help reduce the cost of the works and would increase the potential output at the powerhouses. Concurrently, there would be some reduction in the power output of the present and prospective high head plants at Niagara.

Also of importance to the power entities is the regulation of flows during the winter months. Velocities which would ensure the formation of an ice cover can be obtained by limiting the flow and by enlarging the channel, the final determination being based on economic considerations. These considerations resulted in the selection of 220,000 cubic feet per second as the maximum outflow from Lake Ontario during the ice-forming period in the International Rapids section. Also, since both power entities have greater load requirements during winter months, the minimum regulated flows during the winter should be greater than those for the summer. The entities

indicated the order of importance of the winter months from a power load standpoint as follows: December - 100 per cent; November - 99 per cent; January - 98 per cent; February - 96 per cent; October - 95 per cent; March and September - 90 per cent.

Power Interests Downstream

These interests incorporate the existing developments of the Quebec Hydro-Electric Commission at Cedars and Beauharnois, for further potential between Lakes St. Francis and St. Louis and the undeveloped power of the Lachine reach of the river. As in the case of the upstream power interests the regulation of flows during the winter months is important and it is desirable that river velocities be such as to permit the formation and maintenance of an ice cover above the different power projects. It is essential, therefore, that the regulation upstream should not aggravate operating conditions for the downstream power interests.

At present, plans are being studied for a power development in the Lachine section. To assure the safe and dependable operation of such a development, the discharge capacity of the restrictive reaches above the Lachine Rapids must be enlarged such that the resulting velocities will permit the formation of an ice cover. For this reason, one of the factors that must be considered in the regulation of Lake Ontario is the resulting outflow from Lake St. Louis during the ice-forming period in the Lachine area which generally occurs two to three weeks earlier than in the International Rapids section.

With the advent of winter, the water flowing from Lake Ontario gradually cools as it proceeds downstream. The result is that the freeze-up in the Montreal area is usually two to three weeks earlier in the season than in the International Rapids section. Because of this it will be necessary to limit the regulated outflow from Lake Ontario during the latter half of December or during the month of January.

Studies made by the downstream power interests indicate that for an economical power development in the Lachine section the maximum outflow from Lake St. Louis during the ice-forming period should not exceed 240,000 cubic feet per second because of the large quantity of channel enlargements involved. In this connection the Canadian Government has redesigned the 27-foot seaway canal from Lake St. Louis to Laprairie Basin so as to by-pass 40,000 cubic feet per second through this channel during the non-navigation season. This will permit a maximum outflow from Lake St. Louis of 280,000 cubic feet per second during the ice-forming period.

Methods of Approach to Regulation

Artificial control of the outflows and levels of Lake Ontario must follow some preconceived rule, the effectiveness of which may best be tested by applying it to conditions of supply to the lake over the period for which records are available, 1860-1954. If the rule accomplishes what is desired over this long period, it may be assumed it will meet the requirements under similar supplies in the future. Under operation, the rules indicate definite action to be taken with various conditions of lake levels and supplies. Specific rules are necessary in order to safeguard the various interests involved in the regulation problem. Also, such rules are necessary to enable the several interests to assess the probable effects any plan will have under future

operation by studying its effects when tested under supply conditions of the past.

Additional studies and facilities will be required to permit the prediction of weather and of supplies to the lake with sufficient accuracy for satisfactory regulation of the lake on the basis of predicted supplies alone. However, rules derived for the regulation of the lake necessarily incorporate an estimate of probable future supplies and include sufficient latitude to allow for the maximum deviations from the estimates that have occurred during the period of record. These estimates may not be immediately obvious, being implicit in the rules themselves as determined through trial and error adaptation to past supply conditions.

In its studies the Board investigated four approaches to the problem of regulating the lake levels and discharges. These approaches were designated the A, B, C and S series. Although the bases for the four approaches are similar, they have evolved differently in their methods of solving particular problems.

The "A" approach to the studies of regulation of Lake Ontario consists of a system of graphical rule curves from which the regulated discharge to be allowed in any period of a month or half-month is determined by the level of Lake Ontario at the end of the previous period. The rule curves include certain minimum rates and maximum limitations on the allowable discharge. The discharge determined from the curves is modified by a correction which is based on the variation in supply of the preceding period from the long-term mean for the same period. This correction factor has a small effect on any one month but in the routing procedure its results are cumulative throughout the open-water season. The observance of the criteria and estimates of future supplies are embodied in the rule curves by a process of trial and error.

The "B" approach involves the use of two devices in the procedure for determining regulated outflows: a method of estimating supply to the lake for any regulation period; and, a series of objective end-of-period lake elevations, one for each regulation period in the year. The regulation periods used in this approach to date are the calendar months, except for April and December where half-month periods are used. In addition to the two devices mentioned, discharge limitations, expressed as a function of the lake level, are incorporated to limit channel enlargements in the International Rapids reach and to avoid aggravation of downstream conditions.

The estimated supply for any regulation period is determined by multiplying the supply of the preceding period by the ratio of the long-term mean supplies of the current period to the preceding period. The objective end-of-period lake elevations are derived from past records by trial and error. They are derived to have values such that, with maximum deviation of actual supplies from those estimated, the criteria will still be met over the period of record. The estimated supplies, the objective elevations and the discharge limitations, are incorporated in a set of rule curves, one for each regulating period.

The approach designated as the "C" series is basically very similar to that of the "B" series, with the main differences being in the method of estimating supply to the lake, the use of a longer period of estimating future supplies (the forecast period), the use of variable objective end-of-period lake elevations, and the method of incorporating into the rule curves discharge limitations to give approximately preproject flows during critical months on Lake St. Louis and downstream.

In the "C" series the supply is correlated with the "routed preproject discharge" to provide an index of the probable supply for the current regulation period. The "routed preproject discharge" may be obtained by routing supplies through Lake Ontario using the 1955 open-water stage-discharge relationship for the gauge at Oswego, New York.

As in the "B" series, there is an attempt made to discharge sufficient water so that an objective lake elevation may be met at the end of each of the forecast periods by making adjustments to the regulated outflow at the end of each regulating period of one month or one-half month. In the "C" series there are three forecast periods; January to June; July to September; and October to December. It was found by trial and error over the past period of record that these objective elevations could be varied according to conditions of long-term prospective supplies. That is, in times of high supplies, the objective elevations would be low; whereas, in times of low supplies, the objective elevations would be high. In this manner, efficient use is made of the storage on Lake Ontario as well as recognizing the effect of natural storage in the upper lakes.

A similar discharge limitation to that outlined for the "B" series is incorporated in the method to limit channel excavations in the International Rapids section. Also, for the critical months on Lake St. Louis this approach includes maximum and minimum discharge limitations, which, being related to the calculated natural discharge from Lake Ontario at the end of the preceding months by reference to the records for the past 95 years, give assurance that the regulated discharge will not vary appreciably from that which would occur under natural conditions. A further flow limitation during the last half of December for downstream power development is related to the flow of the Ottawa River on the 15th of December. As in the "A" and "B" series, the supply estimates, design levels and discharge limitations are incorporated in rule curves for each period of a month or half-month.

The initial study of the "S" series involved the use of a power rule curve for maximum continuous power with a drawdown of 1.7 feet, and estimation of the minimum expected supply for the current month on the basis of the supply for the past month. The scheduled release consisted of the predicted minimum supply plus the storage available above the end-of-the-month elevation of the power rule curve. Maximum seasonal outflows were established for all months and minimum releases were specified when the lake elevation exceeded 247.0 or 247.5.

After adoption of a seasonal distribution of outflows for power, consideration was given to development of an index of probable seasonal supplies. Lake Erie mean monthly elevations were used twice a year to classify the probable supply for the succeeding period as one of three categories; low, medium or high. Objective end-of-month elevations were established for each category. The rule curves were designed for average conditions of each category to produce monthly mean outflows in the shape of the desired power load curve, taking into consideration powerhouse forebay elevations. Limiting release rates were specified for each month in each of the three categories in terms of discharge, with additional limitations to satisfy downstream requirements and to control the amount of channel excavations. The previous month's supply was used as an index of the current month's supply on a basis of average conditions. Variations in monthly mean outflows were limited, and outflows were adjusted for deviation of actual from assumed supply within the monthly period.

ıterim Studies

Early in 1955 the need of the navigation interests and the power entities or advice as to what the critical water profiles under regulation would be in he International reach of the St. Lawrence River became urgent. On the remise that computation based on 48 years of record, that is from 1905 to 952, which included the extreme low water of the 1930's and the high water f 1952, would provide sufficient data to define a suitable range of stage on he lake, interim studies were prepared on that basis for the consideration f the Commission in March 1955. Because of time limitations and to insure omparable results these studies were based on one approach, the "A" rocedure.

Trial plans were selected having the following lower and upper limits of ake Ontario elevations: 244.0 and 248.8; 244.0 and 248.0; 243.0 and 247.0 USLS 1935 datum). These trial plans were selected on the basis that the evels and ranges in stage bracketed the limits of regulated lake levels sug-ested by the various interests in written and oral submissions at Commission earings in 1952 and 1953 on Lake Ontario levels.

The requirements of the various interests are not compatible in some nstances. The interests of shore property are served best by a general owering of high lake levels while the interests of power are best served by naintenance of generally higher stages. Navigation is benefited by generally igher stages particularly at low and medium levels.

In developing a plan of regulation which will benefit shore property on ake Ontario, consideration must be given to the duration of intermediate and igh levels, as well as the maximum level that is reached under the plan. Four trial plans were developed within the three ranges of regulated stage elected for study. Two of the trial plans were selected within the range 44.0 to 248.8 in order to develop more fully the capabilities of regulation vithin those limits particularly with respect to shore property and power. The criteria to be satisfied in the interim studies were essentially the same s those finally approved by the governments in December 1955, except as to ontrolling elevations which were variable in order to evaluate the effects hereof. Briefly stated, the criteria required that:

1) The regulated outflow during the navigation season should not reduce he minimum flow past Montreal Harbour.

2) The regulated winter outflows should be as large as feasible and should e maintained so that the difficulties of winter power operation are minimized.

3) Outflow during the annual spring breakup in Montreal Harbour and in the iver downstream should not be increased by regulation.

4) Outflow during the annual flood discharge from the Ottawa River should ot be increased by regulation.

5) Consistent with other requirements the minimum regulated outflow hould be such as to secure the maximum dependable flow for power.

6) Consistent with other requirements, the maximum regulated outflow hould be maintained as low as possible to reduce channel excavations to a ninimum.

7) The low and high water levels under regulation should be maintained as igh as is consistent with other requirements.

The interim regulation studies showed that damage to shore property would be reduced materially by holding the upper level of regulation to elevation 248.0 and that the maximum range of levels should be at least four feet in order to provide the necessary flexibility of operation and to maintain the basic requirements of downstream interests. It was also indicated that a limiting level on Lake Ontario below 244.0 in the navigation season would result in increased costs and reduced benefits to navigation and power interests in the International reach of the St. Lawrence River and to navigation on Lake Ontario.

As the result of review of these studies the Commission concluded that measures could be taken, having due regard to the interests of all concerned, to regulate the level of Lake Ontario for the benefit of property owners on the shores of the lake in both countries, so as to reduce the extremes of stage which have been experienced in the past. By identical letters dated 17 March 1955, the Governments were informed of the Commission's tentative conclusions that the project works should be operated in accordance with criteria as previously listed plus the following:

8) The regulated monthly mean level of Lake Ontario should not exceed elevation 248.0 with the supplies of the past as adjusted.

9) Under regulation, the frequencies of monthly mean elevations of approximately 247.0 and higher should be less than would have occurred in the past with supplies as adjusted and with 1955 channel conditions in the Galops Rapids section.

10) The regulated level of Lake Ontario on 1 April should not be lower than 244.0. The regulated monthly mean level of the lake from 1 April to 30 November should be maintained at or above elevation 244.0.

11) In the event of supplies in excess of the supplies of the past as adjusted the works should be operated to provide all possible relief to the riparian owners upstream and downstream. In the event of supplies less than the supplies of the past as adjusted the works should be operated to provide all possible relief to navigation and power interests.

Plan 12-A-9

Subsequent to this action of the Commission, public hearings were held in Rochester, New York, and in Toronto, Ontario. At these hearings the criteria and range of stage set out in the 17 March 1955 letters were explained and discussed. Briefs and oral statements were presented by various interests.

In the meantime, the Board initiated studies to develop a plan of regulation to meet the requirements of the criteria and range of stage for the entire period of record, 1860 to 1954. A plan of regulation designated 12-A-9 was developed and presented to the Commission at an executive session on 5 May 1955 in Buffalo, New York.

Plan 12-A-9 was developed by the method of approach designated the "A" series. As a plan of the "A" series, it employs a system of rule curves which determine the regulated outflows for one-month periods, or in the case of April and December, for half-month periods, as a function of the Lake Ontario stage at the beginning of the period. A correction is applied to the regulated discharge as determined from the rule curves.

The plan was considered to satisfy the criteria reasonably well although it as recognized that certain minor adjustments were desirable primarily in onnection with downstream effects. The frequency of low levels on Lake St. ouis and resulting low outflows during the summer months was increased omewhat and the high outflows from Lake St. Louis during the winter months lso occurred too frequently during the winter period 15 December to 31 Iarch. Figure 3 shows the results of the plan for the years 1951 and 1952. s a result of its deliberations on 5 May 1955 the Commission recommended the Governments by identical letters dated 9 May 1955 that the criteria set ut in their letters of 17 March 1955 and the range of elevation, 244 (navigaion season) to 248.0 as nearly as may be, be adopted. It was further recomnended that final design of channel excavations be based on this range and lan of regulation 12-A-9, with the assurance that any adjustments required vould be of a minor nature. The Commission also instructed the Board to ndertake a further review of the plan and its effect on the various ownstream interests and develop such minor adjustments to Plan 12-A-9 as night be necessary to best meet their requirements.

A work group established by the Board made a field reconnaissance and btained the informal views of various interests in the Lake St. Louis-Iontreal Harbour reach of the river. As a result of this reconnaissance, and fter studying the views of the various interests, it was concluded that Plan 2-A-9 provided: (a) levels on Lake St. Louis during the summer months vhich were lower than those under natural conditions and objectionable to he riparian and recreational interests; and (b) flows during the last half of December and during the months of February and March that were, on the verage, higher than those that had been experienced in the past and might esult in an aggravation of the ice problem in the Montreal Harbour area. The work group was informed that Plan 12-A-9 provided flows during the econd half of December which would make development in the Lachine section of the river prohibitive from an economic standpoint, and also provided ncreased flows, on the average, during the period December through April, vhich might aggravate the ice problems.

In consideration of these factors, a plan of regulation, involving minor evisions to Plan 12-A-9, was developed. This plan eliminated many of the bjectionable features of Plan 12-A-9 insofar as downstream interests are oncerned.

Approval by the Governments

overnments' Letters of 3 December 1955

Although the modification of Plan 12-A-9 had not been presented to the Commission or Governments formally, the Governments took into consideraion the results of this plan, particularly in regard to its similarity to Plan 2-A-9 with respect to its critical profiles for channel excavations, and the djustments it contained to satisfy downstream interests. Accordingly, in he Governments' reply of 3 December 1955 to the Commission's letters of May 1955, Plan 12-A-9 was approved for the purpose of calculating critical rofiles and the design of channel excavations only. The Commission's ecommended range of stage of 244.0 (navigation season) to 248.0 as nearly s may be, and the recommended criteria for the operation of the regulatory vorks were approved.

Further, with respect to the adjustments to Plan 12-A-9, the Government of Canada, in a letter dated 3 December 1955, to the Chairman, Canadian Section, International Joint Commission, expressed its concern about the effects which the regulation of Lake Ontario levels might have in the exclusively Canadian section of the river, particularly in relation to the flows during the ice-forming period each year and the flooding hazard in February and March each year in the Montreal area. Also, the Government of Canada informed the Commission that arrangements had been made for the redesign of the new 27-foot canal in the vicinity of Montreal which would allow a flow of 40,000 cubic feet per second to be by-passed from Lake St. Louis to Laprairie Basin through the canal during the non-navigation season.

Following the Governments' replies dated 3 December 1955 to the Commission's recommendations made in the 9 May 1955 letters, the Commission met with its advisers, including the Board, on several occasions and on 2 July 1956, issued a Supplementary Order to its 29 October 1952 Order of Approval This Supplementary Order gave effect to the actions of the Governments with respect to the adoption of criteria, the range of stage and acceptance of Plan 12-A-9 as the basis for calculating the critical water profiles and designing the channel excavations in the river.

Some Remaining Problems

In their letters of 3 December 1955 the Governments of Canada and the United States urged the Commission to continue its studies with a view to perfecting the Plan of Regulation so as to best meet the requirements of all interests both upstream and downstream, within the approved range of elevations and criteria. These studies are proceeding under the direction of the International St. Lawrence River Board of Control.

In the continuation of these regulation studies general refinements in the mechanics of the procedures will need to be considered in addition to the effort to meet more fully the requirements of both upstream and downstream interests. Some of the items to be considered are:

1) Use of shorter regulation period than the month to reduce the lag between the occurrence of abrupt changes in supply and the required response in outflows:

2) In the development of Plan 12-A-9 and its modification minima and maxima flows in increments of 5,000 cubic feet per second were selected. If defining these limits more closely would provide additional benefits to any particular interest or interests, such a refinement could be made with very little difficulty.

3) Adjustment of the slope of some of the rule curves in modified Plan 12-A-9 so as to reduce the large change in regulated outflow called for by small changes in lake level.

4) Coordination of releases from Lake Ontario during critical periods with flows of the Ottawa River so as to make most efficient use of available downstream channel capacities.

5) Duplication of downstream preproject flows during certain critical periods on an actual period to period basis rather than satisfying the downstream criteria on an average basis. This also involves item (4).

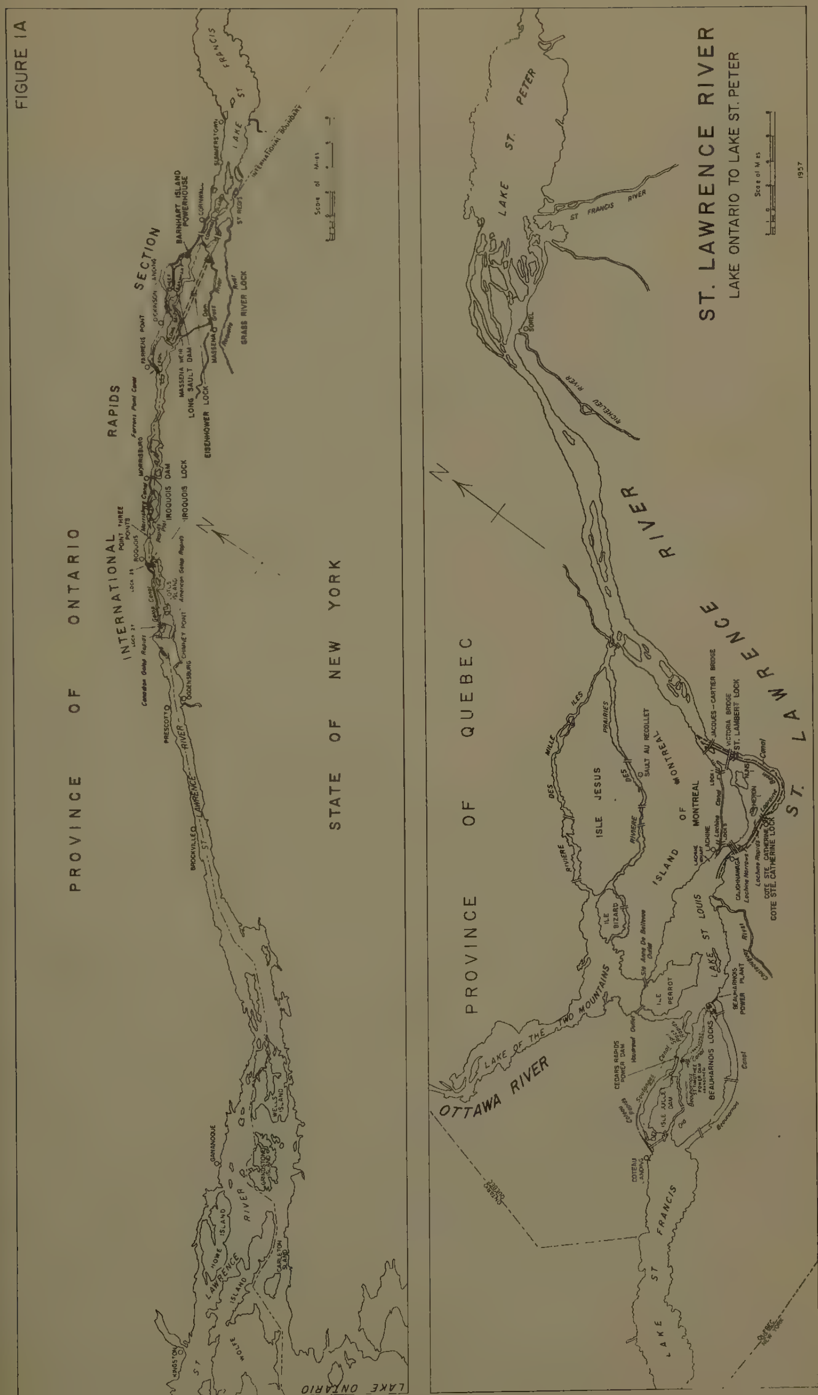
6) Decrease in the magnitude and frequency of large variations in regulated flow from Lake Ontario from month to month.

7) Reduction of the frequency of high outflows during the late winter and early spring months and of low outflows during the late summer and early fall months.

ACKNOWLEDGMENT

This paper has been based on the studies and reports of the International Lake Ontario Board of Engineers.


FIGURE I
GREAT LAKES DRAINAGE BASIN
Scale of Miles
50 0 50 100
1957
LAKE SUPERIOR
LAKE MICHIGAN
LAKE HURON
LAKE ERIE
LAKE ONTARIO
Georgian Bay
OGOKI DIVERSION
LONG LAKE DIVERSION
OGOKI RIVER
KENOGAMI RIVER
LONG LAKE
LAKE NIPIGON
PORT ARTHUR
FORT WILLIAM
DULUTH
INTERNATIONAL BOUNDARY
SAULT STE. MARIE
ST MARYS RIVER
STRAITS OF MACKINAC
CHICAGO
ILLINOIS & MICHIGAN SHIP CANAL
CHICAGO SANITARY & SHIP CANAL
ST CLAIR RIVER
LAKE ST CLAIR
DETROIT
DETROIT RIVER
WINDSOR
TORONTO
HAMILTON
PORT DALHOUSIE
WELLAND CANAL
NIAGARA FALLS
BUFFALO
ROCHESTER
OSWEGO
KINGSTON
THOUSAND ISLANDS SECTION
OGDENSBURG
INTERNATIONAL SECTION
MASSENA
INTERNATIONAL RAPIDS
CORNWALL
LAKE ST FRANCIS
LAKE ST LOUIS
MONTREAL
LAKE ST PETER
OTTAWA
OTTAWA RIVER



ST. LAWRENCE RIVER
LAKE ONTARIO TO LAKE ST. PETER
1957
PROVINCE OF ONTARIO
STATE OF NEW YORK
PROVINCE OF QUEBEC
INTERNATIONAL RAPIDS SECTION
LAKE ONTARIO
ST. LAWRENCE RIVER
LAKE ST. FRANCIS
LAKE ST. LOUIS
LAKE ST. PETER
LAKE OF THE TWO MOUNTAINS
OTTAWA RIVER
RICHELIEU RIVER
ST. FRANCIS RIVER
KINGSTON
GANANOQUE
BROCKVILLE
PRESCOTT
OGDENSBURG
MORRISBURG
MASSENA
CORNWALL
SOREL
ISLAND OF MONTREAL
ISLE JESUS
ILE BIZARD
ILE PERROT
WOLFE ISLAND
HOWE ISLAND
WELLS ISLAND
GRINDSTONE ISLAND
CARLETON ISLAND
IROQUOIS DAM
IROQUOIS LOCK
LONG SAULT DAM
MASSENA WEIR
EISENHOWER LOCK
GRASS RIVER LOCK
BARNHART ISLAND POWERHOUSE
INTERNATIONAL BOUNDARY
BEAUHARNOIS LOCKS
BEAUHARNOIS POWER PLANT
CEDARS RAPIDS POWER DAM
ISLE JULIE DAM
COTE STE. CATHERINE LOCK
ST. LAMBERT LOCK
VICTORIA BRIDGE
JACQUES-CARTIER BRIDGE
SAULT AU RECOLLET
RIVIERE DES PRAIRIES
RIVIERE DES MILLE ILES

FIGURE IA

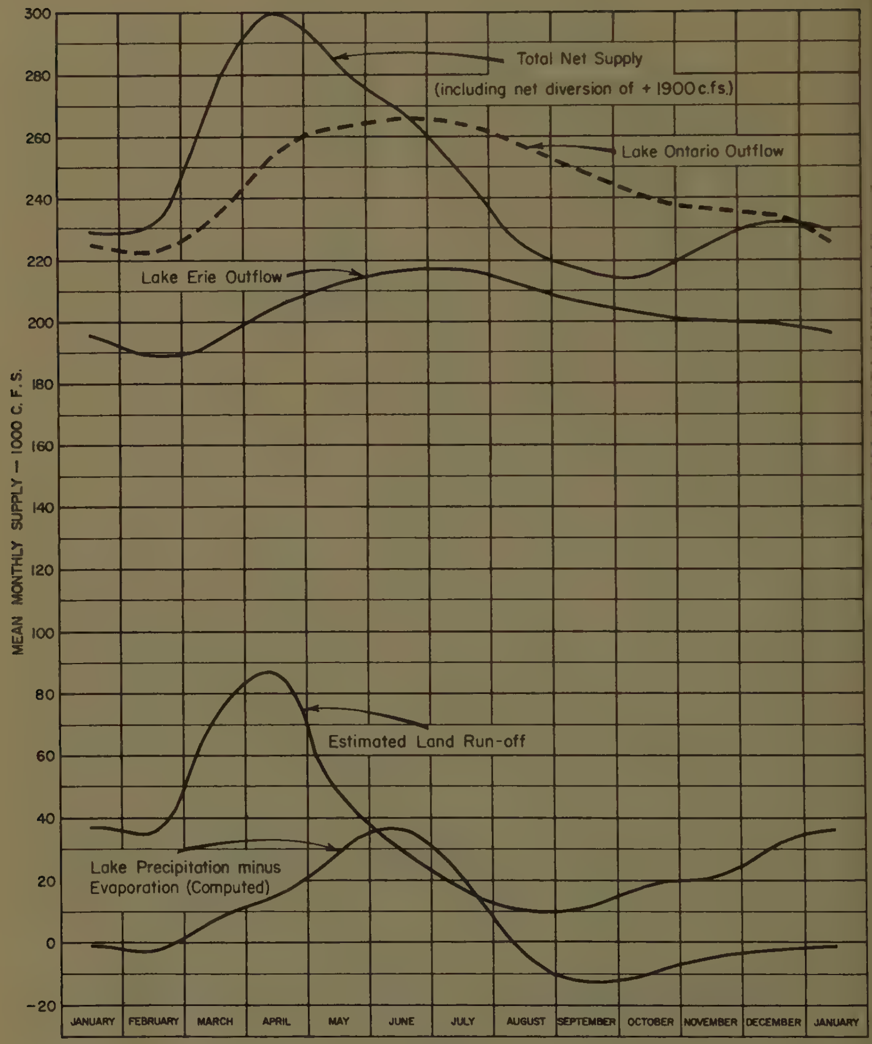


FIGURE 2

AVERAGE VARIATION IN SUPPLY TO LAKE ONTARIO

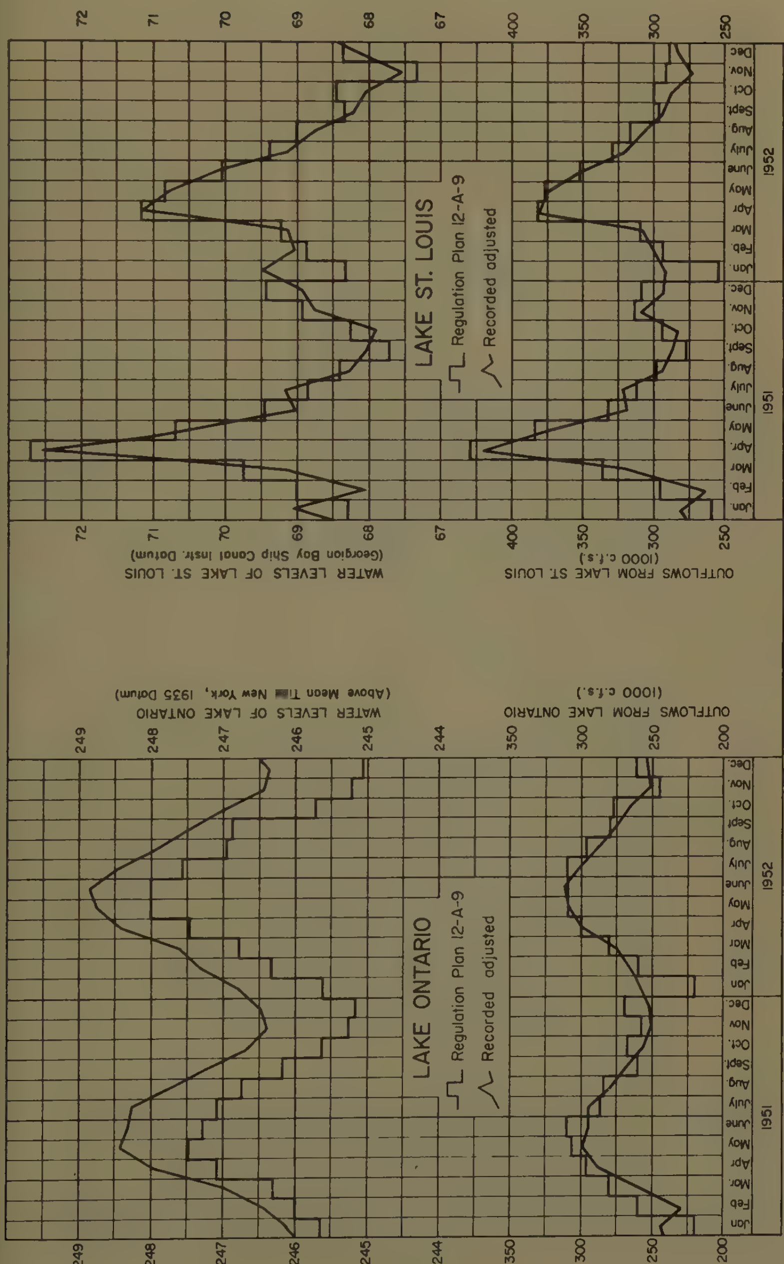


FIGURE 3

EFFECTS OF REGULATION PLAN 12-A-9 FOR THE YEARS 1951 AND 1952.

Journal of the
HYDRAULICS DIVISION
Proceedings of the American Society of Civil Engineers

NORTHEASTERN FLOODS OF 1955: METEOROLOGY OF THE FLOODS[a]

Charles S. Gilman[1] and Kendall R. Peterson[2]
(Proc. Paper 1661)

EXPLANATORY STATEMENT

The year 1955 produced a series of notable storms and floods that struck the northeastern states. Two of the storms were the results of hurricanes and occurred in October. The symposium of three papers covers (1) the meteorological aspects of the storms, (2) the phenomenal discharges, and (3) the effect of the storms and floods on the hydrologic criteria used by the Corps of Engineers in the design of flood control structures.

The first paper concerning meteorology presents some of the physical reasons for the occurrence of the rainstorms. The rain-producing and energy-producing processes of hurricanes are described. Also considered are the energy sources, the pressure distribution accompanying the release of rain, and the volume of water vapor carried into the region by the moist, warm currents.

The second paper briefly describes the floods of August and October 1955. Outstanding peak discharges are listed for selected gaging stations, and a comparison made with the rainfall causing them. Also, a comparison is made with past floods. Some indication of the frequency of the floods is presented.

The third paper describes the effect of the 1955 storms and floods on (1) items pertaining to derivation of synthetic design floods, such as depth-area-duration rainfall relationships, unit hydrographs, and infiltration losses; (2) flood frequencies; (3) volume of runoff as it affects reservoir storage capacity and regulation procedures; (4) method of transposing the storms to unaffected areas in New England; and (5) the design capacity of pumping stations for local protection projects.

Note: Discussion open until November 1, 1958. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. Paper 1661 is part of the copyrighted Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers, Vol. 84, No. HY 3, June, 1958.

a. Presented at meeting of the Hydraulics Div., ASCE, Massachusetts Inst. of Technology, Cambridge, Mass., August, 1957.

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ABSTRACT

This paper, one of three of a symposium, is concerned with physical reasons for the occurrence of the rainfall. Part of the explanation is the connection between rain-producing and energy-producing processes; another explanation is the inertia of the winds. A series of maps is presented which illustrate these processes.

Three large rainstorms occurred in the northeastern United States in August and October 1955. The purpose of this paper is to present some of the physical reasons for the occurrence of these rainstorms.

Copious rainfall occurs in the atmosphere when there is a large-scale upward motion of air. This upward motion lifts some of the air in the lower layers to upper layers, where, under the reduced pressures aloft, the temperature falls, lowering the capacity of the air to hold water. The energy conversion processes in the atmosphere are also intimately connected with large-scale upward and downward motion. Potential and thermal energy are converted to kinetic energy when warm air ascends and cold air descends so that there is a close connection between the rain-producing processes and the energy-producing processes. But the elementary physical considerations also show that in order for the winds to increase in speed, that is, for the kinetic energy in the atmosphere to be increased, it is necessary that air blow from higher pressure toward lower pressure. This happens both in the lower atmosphere and in the upper atmosphere nearly simultaneously. So the energy-producing processes, coming ultimately from temperature contrast or the release of latent heat, have an intermediate step which involves the modification of pressure patterns. The greatest amount of ascent occurs not in the center of the pressure formation, but in almost all cases some distance downstream from the pressure center. Similarly the greatest amount of descending air occurs ahead of high pressure systems. These effects are complicated by the rotation of the earth. In the Northern Hemisphere the air tends to curve toward the right in the absence of a pressure gradient.

Figs. 1, 2, and 3 illustrate some typical pressure formations that are associated with heavy rainfall. The light lines are surface isobars which represent pressure gradient shear, curvature, and a combination of both shear and curvature. The heavy lines are trajectories of air parcels passing through the isobars. The numbers within the areas formed by the trajectories indicate the percentage change of area.

However, there are some cases of copious rains that do not seem to be intimately connected with kinetic energy-producing processes. For example, many of the heaviest rains in connection with hurricanes occur during the dying stage of the wind system. There are numerous examples of this which have occurred in hurricanes entering the United States during the past years. Here the inertia of large-scale rapidly moving air currents frequently carries them into regions where the prevailing pressure distribution cannot support the continuance of such a fast-moving current. In these instances the piling up of the fast-moving air against the slower-moving air already in the region causes it to rise, thus giving the heavy rain.

Now going back to the rains associated with kinetic energy production, there are in general two kinds of situations that give the warm air ascent-cold

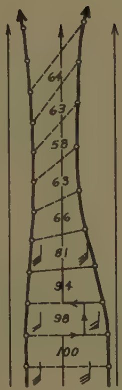

Fig. 1. Change of Area Between Two Trajectories for Shear in Surface Isobars. Numbers indicate percentages of original area.

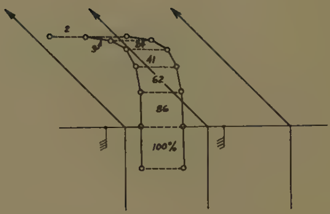

Fig. 2. Change of Area Between Two Trajectories for Sharp Curvature in Surface Isobars. Numbers indicate percentages of original area.

Fig. 3. Surface Wind Distribution and Location of Maximum Rainfall (region A) in a Typical Low Pressure System.

air descent necessary to produce large amounts of kinetic energy, that is, winds. The first is connected with the ordinary frontal cyclone when cold air from regions far north of temperate latitudes comes into close proximity of warm air currents coming from tropical regions far south. The second situation occurs when the atmosphere aloft at elevations from 20,000 to 40,000 feet is comparatively cold, while the atmosphere in the first three to five thousand feet is quite moist. Then, if there is an initial perturbation, it will lift some of this surface air to the higher atmosphere (20,000-40,000 feet). The release of latent heat will be sufficient to produce a lessening of the cooling in this rising air, so that when it arrives at these upper levels, it will be warmer than the air over surrounding regions. The first of these conditions is illustrated by our regular or typical storm in winter in the United States, while the latter is illustrated by the tropical hurricane and to a limited areal extent, by the thunderstorm.

In order to maintain a hurricane, air from a large region in the lower atmosphere is drawn into the center of the storm, rises, releasing its latent heat, and then spreads out in the upper atmosphere. The amount of convergence of the air in the lowest atmosphere is very large. Figs. 4 and 5 show schematically the change of shape of an initially cubic mass of air when sufficient upward motion takes place to cause 2 and 5 inches of rain, respectively. The bases of the cubes are at the surface while the tops are at about 40,000 feet. In Fig. 4, for 2 inches of rain, the area at the surface has decreased to 1/10 while in Fig. 5, for 5 inches of rain, the surface area has decreased to 1/100 its original area. Further computations show that air from something like eight or ten times the area within a 60-mile radius of a large hurricane is drawn into the low level circulation every 24 hours so that during the course of a week in the life history of one of these storms air from an area equal to a large percentage of the area of the tropical Atlantic Ocean might be drawn into the storm. Similar considerations hold on a smaller time scale for large rainstorms in connection with temperate latitude systems.

Now, when a hurricane moves from tropical regions to middle latitudes, two modifications can take place. First, the hurricane can encounter colder air along its path. Thus, the air moving around the center, say around the east side moving toward the north, might encounter a colder air mass, produce a trough of low-pressure air, and the most copious rainfall might then spread out several hundred miles to the north of the system. Second, as a hurricane moves over land its pressure system weakens, so the condition described earlier occurs where the strong current of air to the right of the pressure center moves into a region of relatively flat pressure gradient. Then this air tends to curve around to the right and produce a piling up of this air against the inert slower-moving air that was already in the region, which can produce very heavy rainfall ahead and to the right of the storm path. Both of these phenomena occur quite often as hurricanes move into the United States.

Now, the total volume of precipitation that falls in either of these situations, is determined, to a large extent, by the volume of water vapor entering into the proximity of the storm center, so that volumetrically, at least, we can foresee to a certain extent, the amount of precipitation that will fall by measuring the inflow even several hundred miles from where it will occur. Here one may comment on the tremendous distances moved by air in some of our major storms. For example, in one day air near the surface can move from the vicinity of the Bahama Islands to Pennsylvania or southern

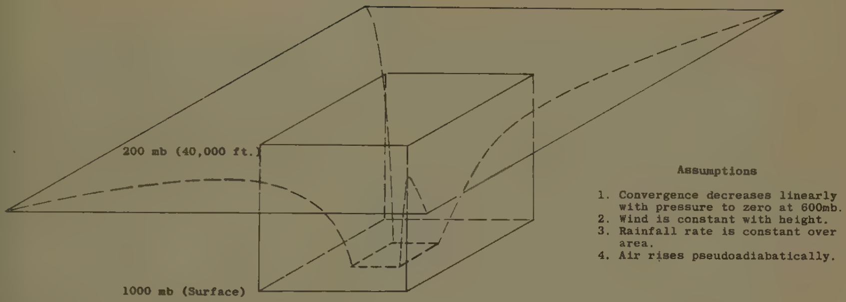

Fig. 4. Change of Shape of Mass of Air Necessary for 2 Inches of Precipitation at Surface Temperature of 70°F.

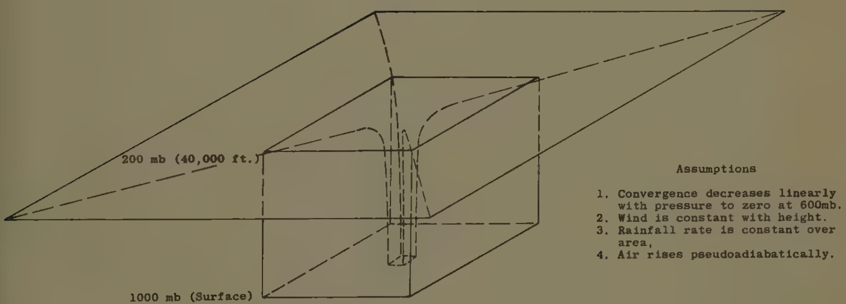

Fig. 5. Change of Shape of Mass of Air Necessary for 5 Inches of Precipitation at Surface Temperature of 70°F.

New England, even in a typical situation. The water vapor that falls as rain over New England today was three days ago, in many cases, in the tropical Atlantic Ocean east of the West Indies.

Summarizing, we may consider the physical hydrodynamics of the rain-producing process as composed of the coincidence of two factors. The first is a general characteristic of the pressure field at various levels in a region which will produce general upward motion and the second is the blowing through this region of a current containing a large volume of water vapor. In the discussion of the three storms we will consider the energy sources, the pressure distribution which accompanied the release of rain, and the volume of water vapor carried into the region by the moist, warm current.

Hurricane Connie

Fig. 6 shows the surface weather map for 0130EST August 11, 1955. At this time hurricane Connie was about 180 miles south-southeast of Wilmington, N. C. It will also be noted that there was a general trough running through New England down through Pennsylvania and into northern Virginia. Fig. 7 shows the 24-hour mean trajectories of air parcels at 2000 feet on August 11; also shown are arrows whose length is proportional to the amount of moisture inflow. This moisture inflow is the product of the precipitable water,* the length of the boundary of inflow to the right of the hurricane path, the wind speed normal to this boundary and the time of the inflow (which was 24 hours). The numbers plotted on the left of the station dots are the mean temperature between the surface and 700 mbs (10,000 ft.) in degrees centigrade, and on the right the precipitable water in inches between the surface and 400 mbs (23,000 ft). Adjacent to the heavy arrows are figures which indicate the inflow volume of precipitable water in square mile-inches; the precipitation volume also in square mile-inches, and the efficiency which is defined as the precipitation volume divided by the inflow volume. Fig. 8 is the 24-hour isohyetal map for August 11th from which the precipitation volume was computed. It will be noted that the efficiency—at least as measured by the observed rainfall—in the southeast states is rather low, that is, 41%. At least part of the reason for this was that much of the rainfall fell over the ocean before the hurricane passed onto land. In the New England region the inflow volume is considerably less than in the southeast, however, the efficiency is higher, at least part of the explanation for this being that the precipitation began falling out close to the coast and therefore most of the rainfall was over land and could be measured. These features are evident from the isohyets in Fig. 8.

Fig. 9, the surface weather map for 0130EST August 12th, shows that the hurricane had moved in a general northerly direction and was situated approximately 75 miles from Wilmington, N. C. The trough present on the 11th had moved off the coast and had weakened considerably. In Fig. 10 the boundary of the moisture transport to the right of the hurricane path was 450 miles; the inflow volume was 562,000 square mile-inches, while the precipitation volume was 439,000 square mile-inches, giving an efficiency of 78%. Fig. 11, the 24-hour isohyetal map for August 12th, indicates that the major rainfall

*Precipitable water is defined as the depth of liquid water which would result if all the water vapor in a column of air is precipitated. Thus, it is a measure of the total amount of water in the whole depth of the atmosphere.

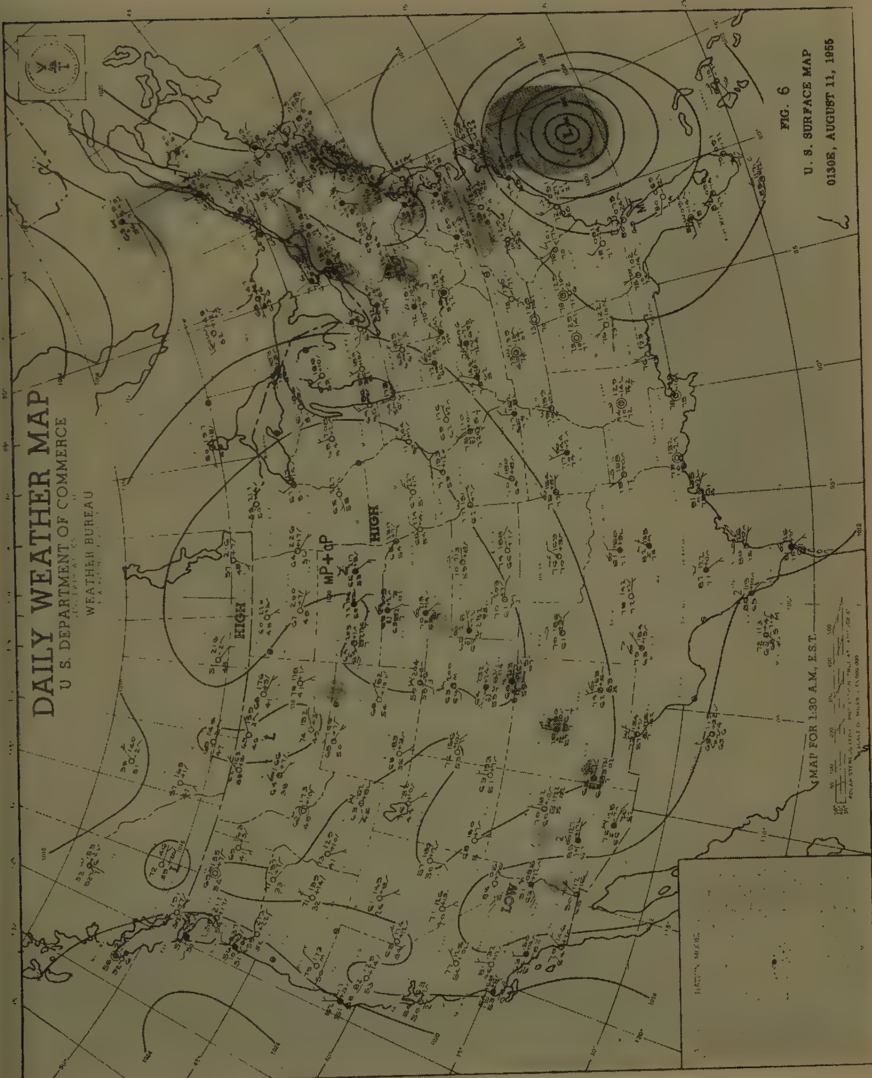

FIG. 6
U. S. SURFACE MAP
0130E, AUGUST 11, 1955

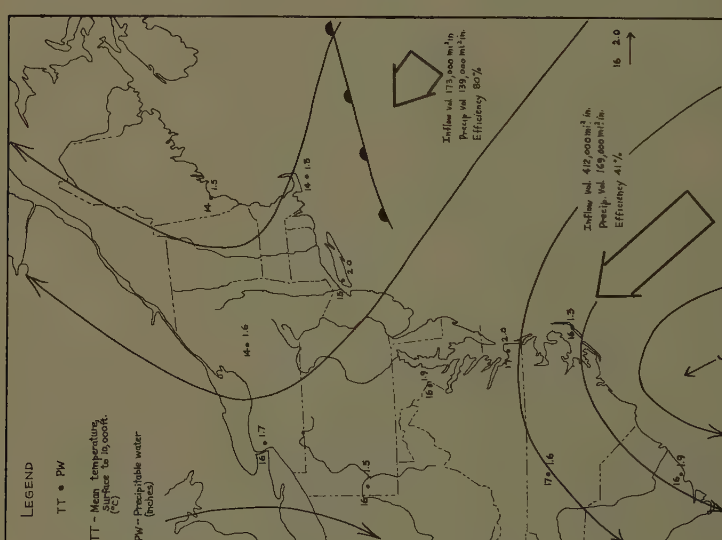

Fig. 7. Mean Trajectories of Air at 2000 Feet. AUGUST 11, 1955

Fig. 8. Isohyetal Map, 24 Hrs. Ending 2400 EST, AUGUST 11, 1955. Precipitation in inches.

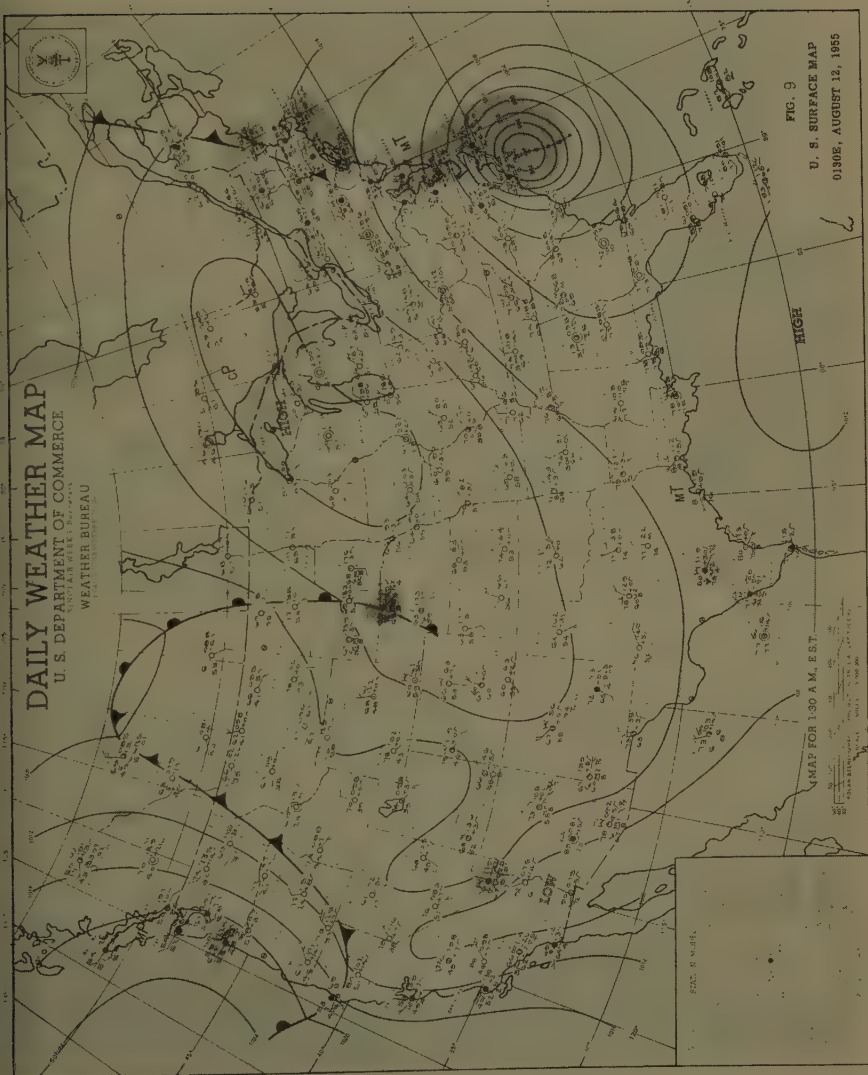

FIG. 9
U. S. SURFACE MAP
0130E, AUGUST 12, 1955

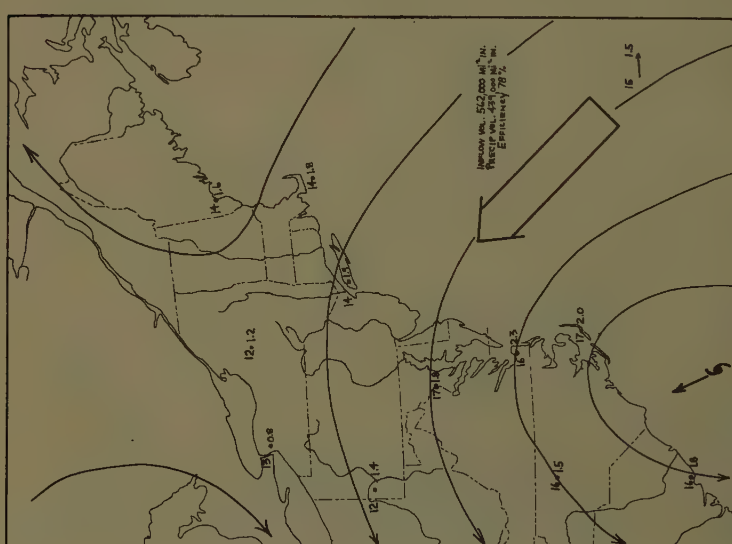

Fig. 10. Mean Trajectories of Air at 2000 Feet AUGUST 12, 1955. See Fig. 7 for legend.

Fig. 11. Isohyetal Map, 24 Hrs. Ending 1900EST AUGUST 12, 1955. Precipitation in inches.

occurred over the Chesapeake Bay area with a secondary maximum over Long Island. It can be seen in this case that some of the rain fell over the ocean where it could not be measured and therefore the efficiency may well be higher than indicated.

Hurricane Connie passed inland in the forenoon of August 12th near Cherry Point, N. C., and by 0130EST August 13th (Fig. 12) Connie was located in eastern Virginia. The mean temperatures on the preceding days had indicated some decrease of temperature outward from the center, but the warm center was more clearly defined on August 13. Fig. 13 also shows the mean trajectories around the hurricane center during the 13th. The computed inflow volume was 518,000 square mile-inches, while the precipitation volume was 393,000 square mile-inches, giving an efficiency of 76%. From Fig. 14 it can be seen hat the precipitation was occurring practically all over land with the exception of some light rainfall off the New Jersey and Long Island coasts. The precipitation pattern had become more widespread with several centers. It is interesting to note that on this day, as well as on the 11th and 12th, precipiation had been occurring in New England with the hurricane center to the southeast and passing off toward the northwest. This rainfall was due first to he weak trough which passed through New England on the 11th and then to the curvature in the isobars around the hurricane during the 12th and 13th. Furthermore, in this storm, the boundary of moist inflow was apparently greater than usual, being 450 miles on the 12th and 400 miles on the 13th. On August 14 the hurricane moved toward the northwest and passed up into the Great Lakes region. The precipitation on this day was very light and no maps are shown. Fig. 15 shows the total storm precipitation between August 11-14. Of particular interest is the heavy rainfall in New England and eastern New York. The dashed line is the track of hurricane Connie, with location at 0730EST and 1930EST shown by circled number and solid dot, respectively.

Hurricane Diane

The next hurricane studied was Diane. This hurricane passed inland in North Carolina at 0730EST August 17th and lost most of its hurricane force almost immediately. However, it maintained its center and slowly curved toward the north and then toward the northeast passing out to sea south of Long Island on August 19th. Although the winds had diminished in this hurricane after passing inland the moisture was still carried along with it and this resulted in heavy rains in the Pennsylvania, New York, and New England region on the 18th and 19th of August. Fig. 16 shows the surface map for 0130EST August 17. At this time hurricane Diane was southeast of Wilmington, N. C., with a weak low-pressure system north of the Great Lakes with a warm front passing through northern New England. The moisture transport into the New England region on the 17th was very light, therefore only slight amounts of rainfall could be expected. The inflow volume on August 17th for hurricane Diane is shown in Fig. 17. This inflow was 353,000 square mile-inches. The precipitation volume as computed from Fig. 18 was 251,000 square mile-inches thereby giving an efficiency value of 71%. It can be seen from Fig. 18 hat the rainfall was spread out over Virginia and North Carolina. Some of he rainfall in this case was falling over the ocean off the North Carolina coast.

The 0130EST surface map for August 18th is shown in Fig. 19. The center

FIG. 12
U. S. SURFACE MAP
0130E, AUGUST 13, 1955

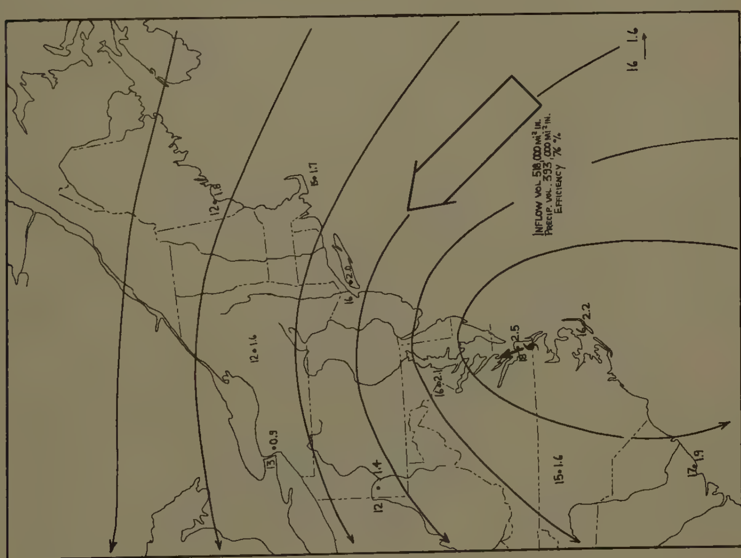

Fig. 13. Mean Trajectories of Air at 2000 Feet AUGUST 13, 1955. See Fig. 7 for legend.

Fig. 14. Isohyetal Map, 24 Hrs. Ending 1900EST AUGUST 13, 1955. Precipitation in inches.

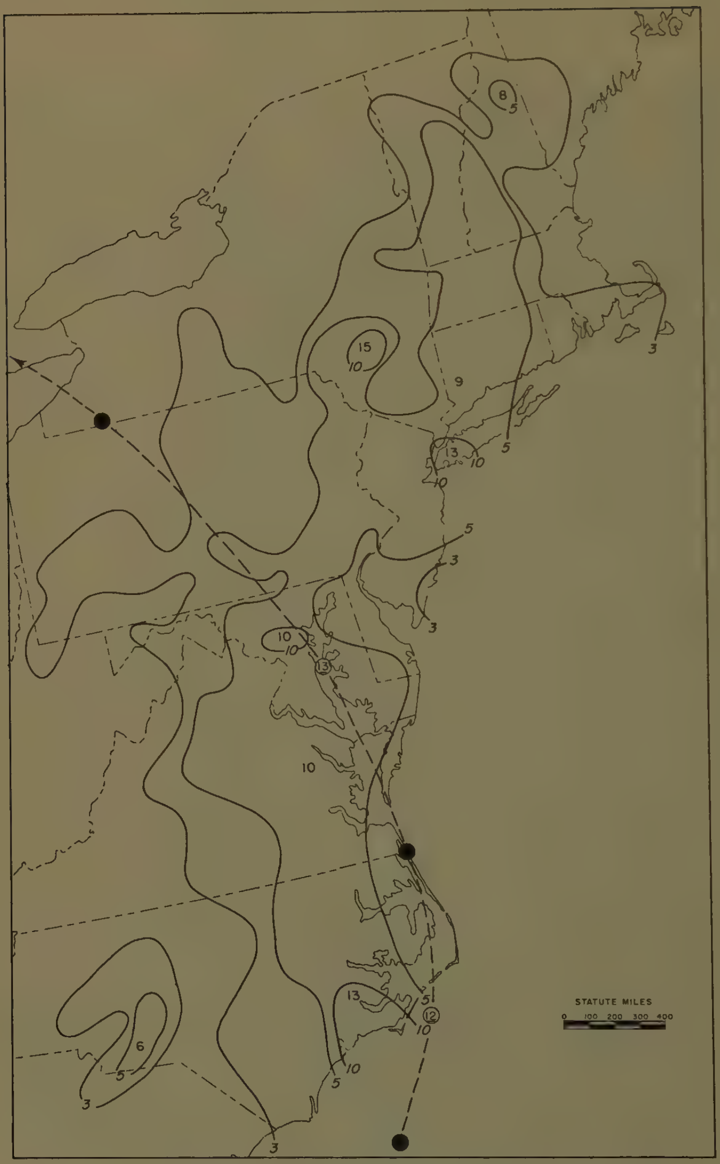

Fig. 15. Total Storm Isohyetal Map, AUGUST 11-14, 1955. Precipitation in inches. Dashed line is hurricane track. Numbered circles are locations of hurricane center at 0730 EST on days indicated.

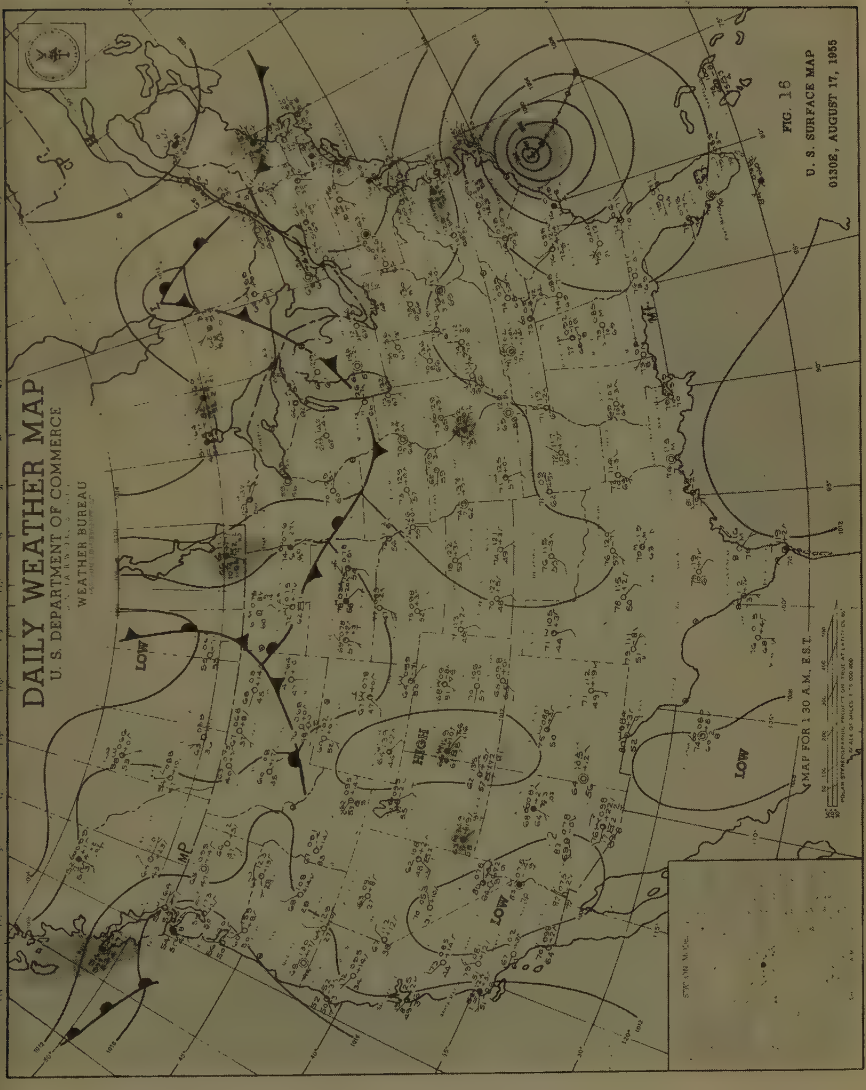

FIG. 16
U. S. SURFACE MAP
0130E, AUGUST 17, 1955

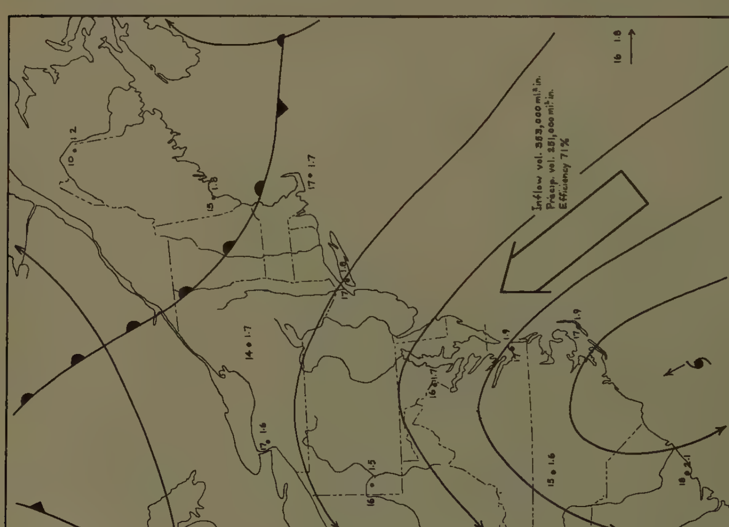

Fig. 17. Mean Trajectories of Air at 2000 Feet
AUGUST 17, 1955. See Fig. 7 for legend.

Fig. 18. Isohyetal Map, 24 Hrs. Ending 1900EST
AUGUST 17, 1955. Precipitation in inches.

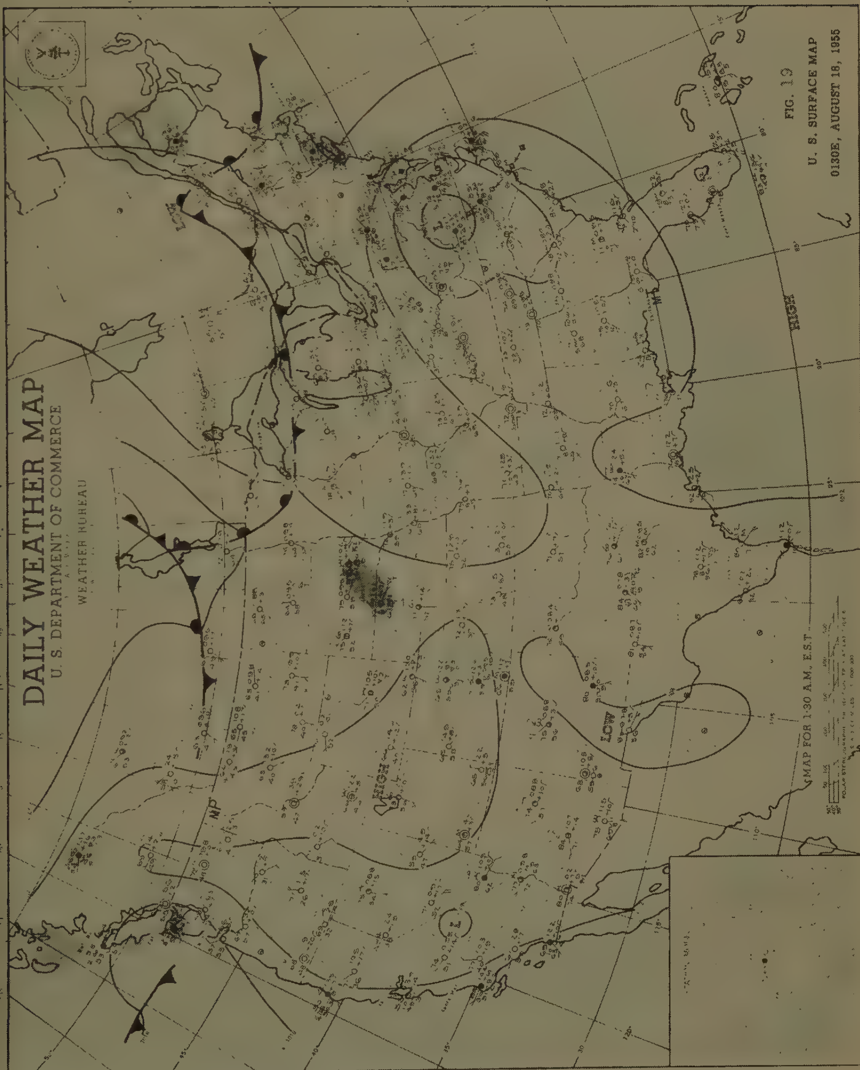

FIG. 19
U. S. SURFACE MAP
0130E, AUGUST 18, 1955

of Diane had moved near Lynchburg, Va., and had begun curving toward the north-northeast. The inflow volume shown in Fig. 20 was 353,000 square mile-inches, while the precipitation volume computed from Fig. 21 was 265,000 square mile-inches which gave an efficiency of 75%. It can be seen that the precipitation on the 18th was much more concentrated than the precipitation on the 17th. However, it is interesting to note that the volume of precipitation observed on the 17th and 18th was practically the same, even considering the fact that some of the rainfall on the 17th had fallen over the ocean while most of the rainfall on the 18th was over land.

This increased concentration of precipitation can be explained by the mean surface to 700 mb (10,000 ft) temperatures shown in Figs. 17 and 20. On August 17th (Fig. 17) these temperatures were 17-18° C. near the hurricane center and decreased very gradually northward. On August 18 (Fig. 20) the mean temperature to the east of the storm center was 20° C. with 17° C. temperatures in the vicinity and with a more rapid decrease of temperature to the north. This indicates that a pocket of warm air had become trapped to the east of the storm center, resulting in a concentrated burst of rainfall to the north of this region since at these higher temperatures the air can hold more moisture. The pocket of warm air is more readily discernible on surface maps which have a better data coverage and are taken at more frequent intervals. These maps (not shown) indicate that the region of warm moist air developed to the east of the storm center shortly before midnight of the 17th but reached a maximum around noon of the 18th.

Fig. 22, the surface map for 0130EST for August 19th, shows the center of hurricane Diane in the vicinity of Philadelphia. It can be noted that the hurricane had maintained a good amount of its circulation, although the winds were far below hurricane force. In Fig. 23 the computed inflow volume was 211,000 square mile-inches while from Fig. 24 the precipitation volume was 134,000 square mile-inches, giving an efficiency of 64%. From the precipitation map it can be seen that a considerable amount of precipitation was falling over the ocean off the Massachusetts coast, therefore giving the low efficiency value. During the 19th, hurricane Diane passed south of New England out to sea and the rainfall ended thereafter. No further maps are shown. Fig. 25 is the total storm map for August 17-19; the dashed curve is the track of hurricane Diane.

The October Rainfall

The third storm studied occurred between October 13th and 18th, 1955. Fig. 26, the surface map for 0130EST October 12th, indicates a stationary front through northern New England with a weak westerly flow over southern New England. From Fig. 27 it can be seen that the trajectories south of the front were coming from land and passing out to sea. Except along the coast, the precipitable water amounts were small and no rainfall was indicated on this day.

By 0130EST October 13th, (Fig. 28) the cold front in the Midwest on the 12th had passed into Ohio and was approaching the Pennsylvania border. There was a weak trough preceding this cold front and running down along the Appalachian Mountains. Fig. 29 shows the trajectories and inflow volume for October 13th. From the trajectories it can be seen that convergence was occurring over Virginia, Pennsylvania, and western New York. Because of the stationary front extending through central New York and south of New England,

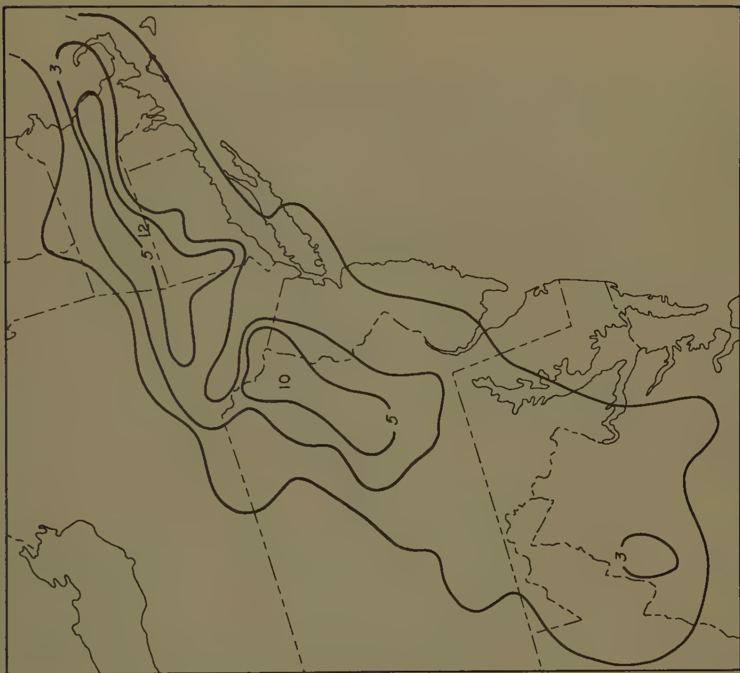

Fig. 21. Isohyetal Map, 24 Hrs. Ending 2400 EST AUGUST 18, 1955. Precipitation in inches.

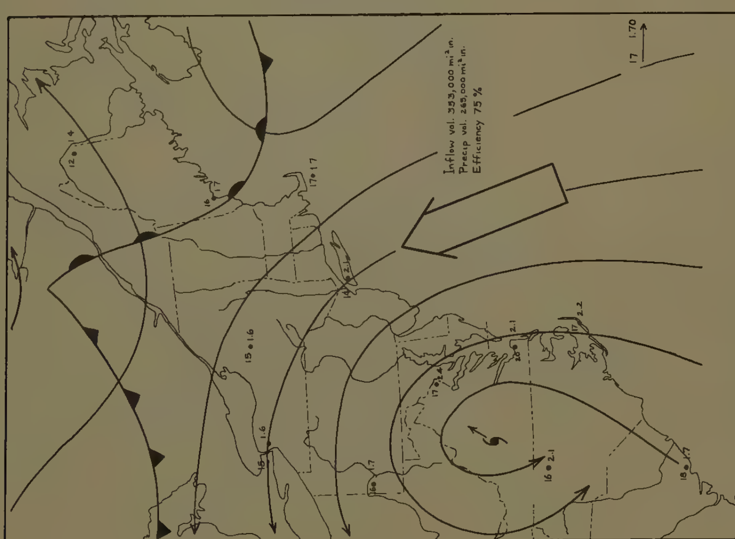

Fig. 20. Mean Trajectories of Air at 2000 Feet AUGUST 18, 1955. See Fig. 7 for legend.

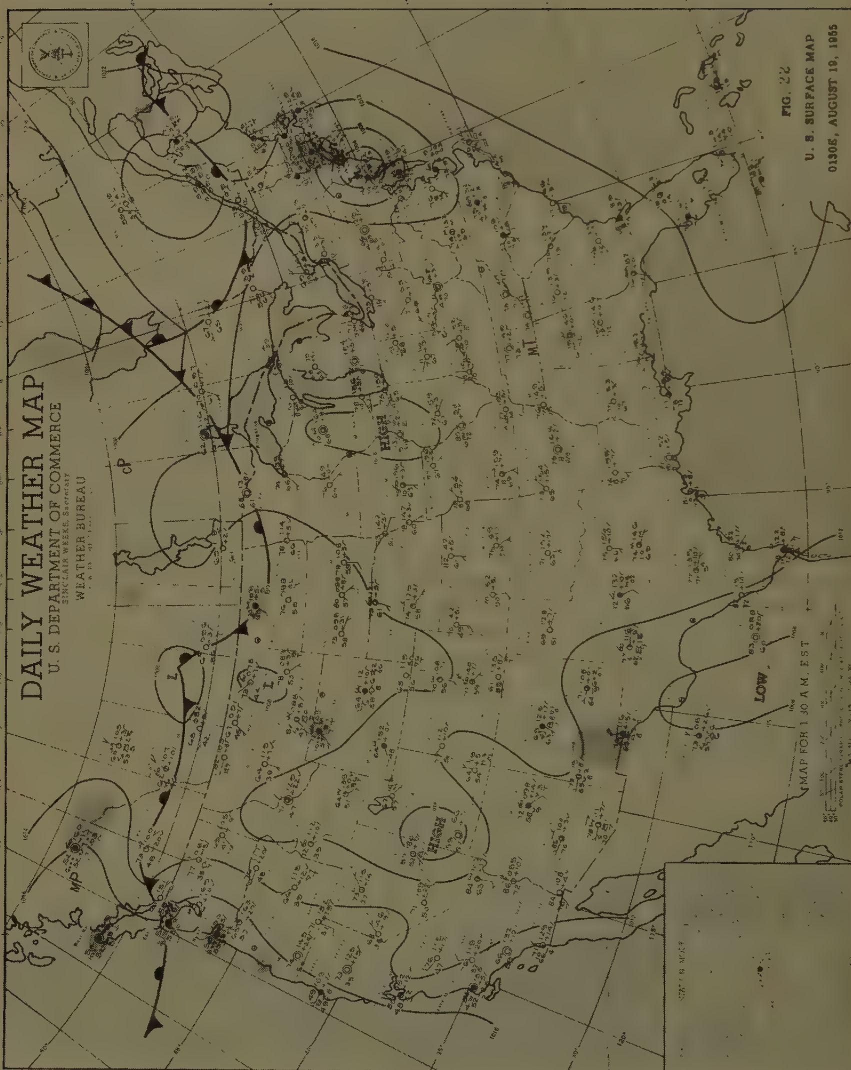

FIG. 22
U. S. SURFACE MAP
0130E, AUGUST 19, 1955

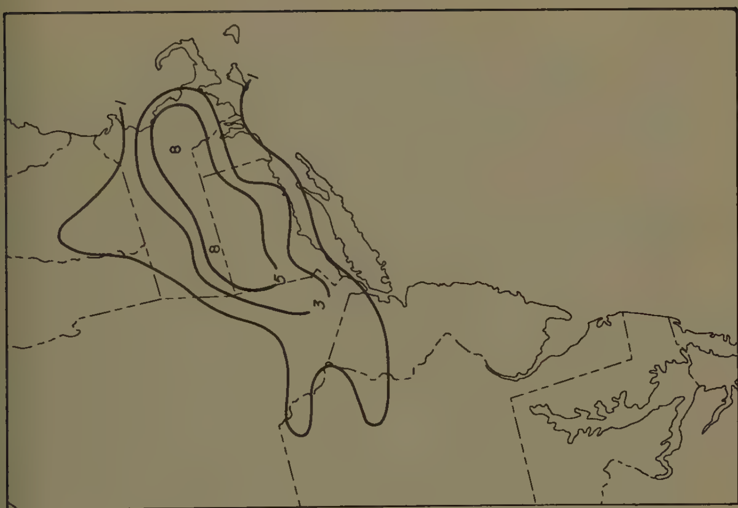

Fig. 24. Isohyetal Map, 24 Hrs. Ending 2400 EST AUGUST 19, 1955. Precipitation in inches.

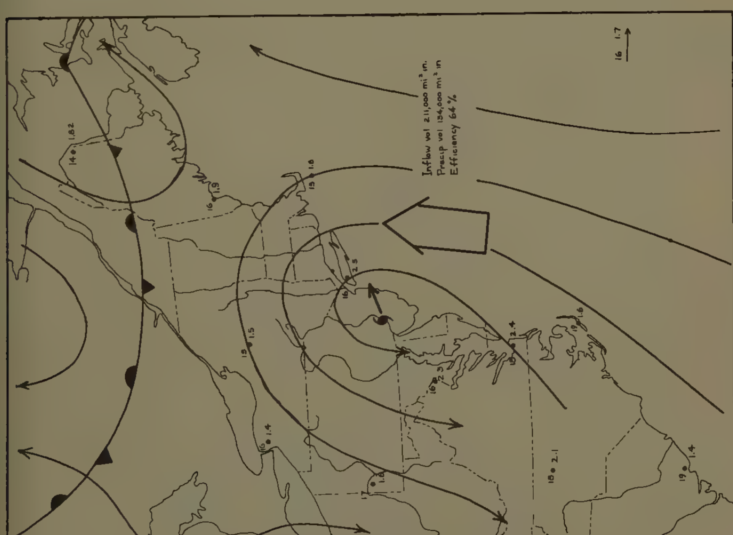

Fig. 23. Mean Trajectories of Air at 2000 Feet AUGUST 19, 1955. See Fig. 7 for legend.

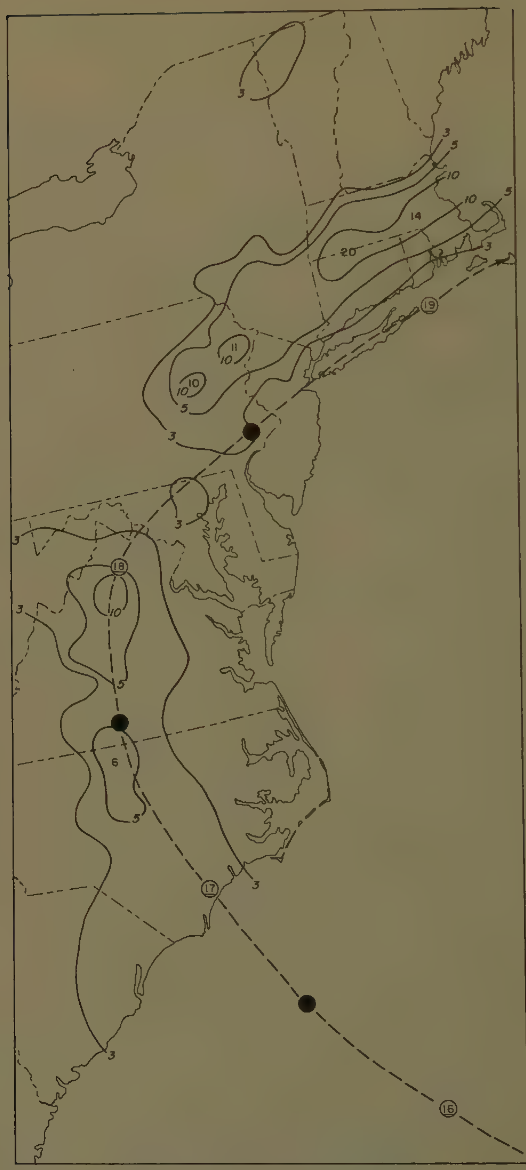

Fig. 25. Total Storm Isohyetal Map, AUGUST 17-19, 1955. Precipitation in inches. Dashed line is hurricane track. Numbered circles are locations of hurricane center at 0730 EST on days indicated.

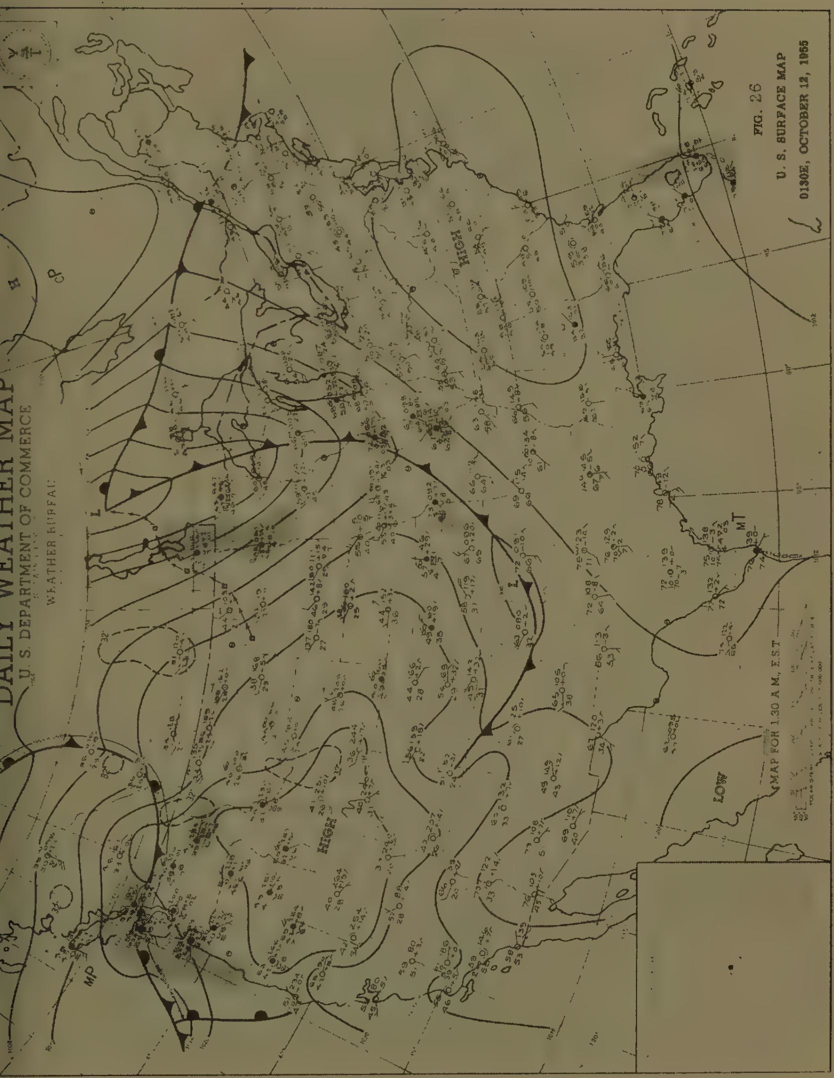

FIG. 26
U. S. SURFACE MAP
0130E, OCTOBER 12, 1955

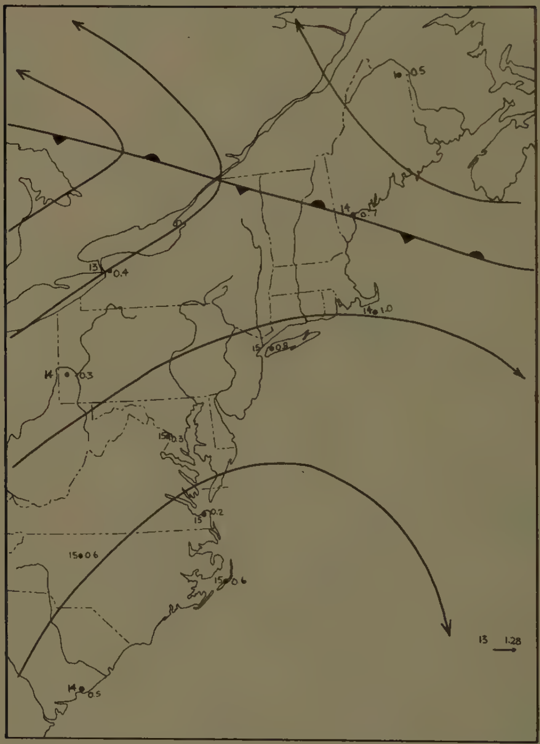

Fig. 27. Mean Trajectories of Air at 2000 Feet OCTOBER 12, 1955. See Fig. 7 for legend.

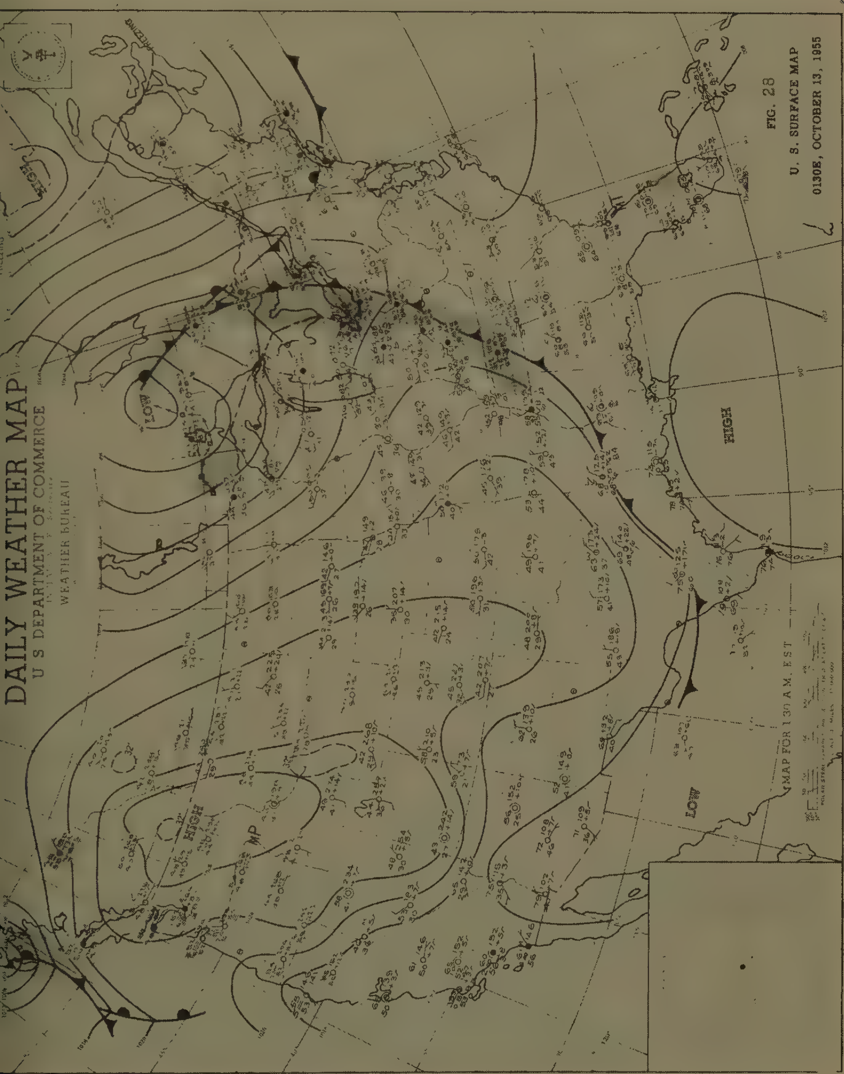

FIG. 28
U. S. SURFACE MAP
0130E, OCTOBER 13, 1955

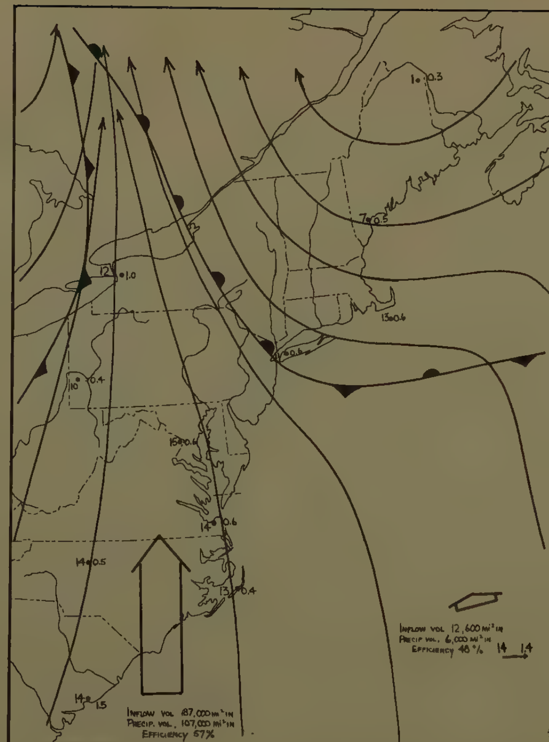

Fig. 29. Mean Trajectories of Air at 2000 Feet OCTOBER 13,1955. See Fig. 7 for legend.

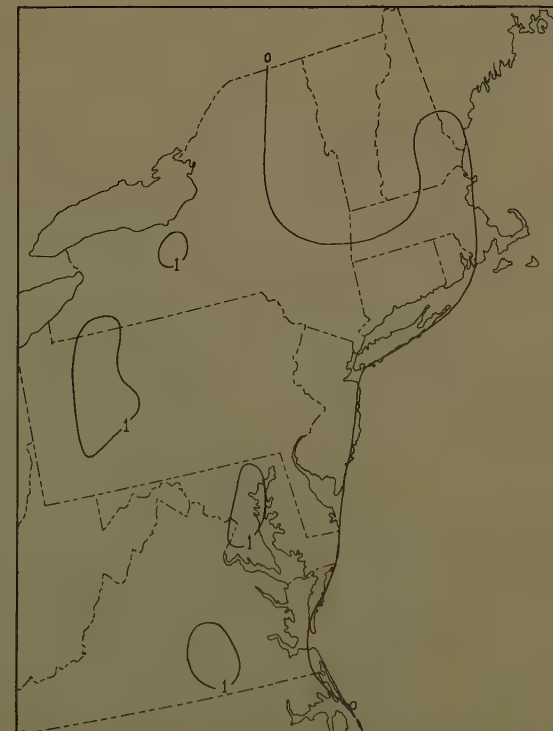

Fig. 30. Isohyetal Map, 24 Hrs. Ending 2400 EST OCTOBER 13, 1955. Precipitation in inches

; shown in Fig. 28, the inflow volume computation was broken down into two
ırts, as shown in Fig. 29. For the western part the inflow volume was
37,000 square mile-inches, while the precipitation volume computed from
ig. 30 was 107,000 square mile-inches, giving an efficiency of 57%. For the
ıstern inflow computation the inflow volume was 12,600 square mile-inches,
e precipitation volume was 6,000 square mile-inches, and the efficiency was
3%. The small efficiencies in both these cases were probably related to the
nall amount of shear and curvature in the isobars.

Fig. 31 shows the surface map for 0130EST October 14. The cold front
ıssed through central New York, Pennsylvania, Virginia, and the Carolinas
ith a stationary front passing east-southeastward through New Jersey out to
ea. A secondary low had developed and was situated southeast of Hatteras,
. C. The fact that this storm was a cold-core system can be seen from the
ean temperatures on October 14 (Fig. 32), as well as on succeeding days.
he trajectories in Fig. 32 indicated convergence in southern New England,
ew Jersey, New York, and Pennsylvania. The inflow volume was 302,000
quare mile-inches, while the precipitation volume from Fig. 33 was 218,000
quare mile-inches, giving an efficiency of 72%. From the precipitation map
can be seen that the pattern was somewhat broken up with most of the rain
ccurring in Pennsylvania, New Jersey, and southern New York, and with rain-
ll amounts greater than one inch moving into southern New England.

Fig. 34 shows the surface map for 0130EST October 15th. Most of the
ontal system had passed off the coast, however a stationary front passed
rough New Jersey and out to sea south of New England. It can be seen that
great many isobars passed north of this front, thereby indicating a consider-
ble amount of moisture transport over New England. Fig. 35 shows the tra-
ctories and inflow volume for October 15th. This inflow volume was
30,000 square mile-inches, while the precipitation volume was 167,000 square
ile-inches, giving an efficiency of 73%. Fig. 36 shows the precipitation map
r October 15th. Here the precipitation was more concentrated than before
ıd the larger amounts had moved into southern New England.

Fig. 37 shows the surface map for 0130EST October 16th. The occluded
ontal system passed from western New York through northern New Jersey
a low-pressure system off the New Jersey coast. The warm front con-
ued south of the New England coast passing out to sea. Fig. 38 shows the
ajectories and inflow volume for the 16th. It can be seen that on this day the
oisture was being transported from a general easterly direction. The tra-
ctories show convergence over central and southern New England with some
nvergence over New York. The inflow volume was 176,000 square mile-
ches, while the precipitation volume from Fig. 39 was 133,000 square mile-
ches, giving an efficiency of 76%. From the precipitation map, Fig. 39, it
ın be seen that moderate rainfall was continuing over the southern New
ıgland region with some moderate and lighter rainfall over central and
ıstern New York.

Fig. 40 shows the surface map for 0130EST October 17th. The circulation
ıs weakened somewhat with the front still passing south of the New England
ast. In Fig. 41 the moisture inflow was coming from the east as it was on
e 16th and amounted to 87,500 square mile-inches. The trajectories indicat-
a convergent area over central and northern New England; the precipitation
lume from Fig. 42 was 52,000 square mile-inches, giving an efficiency of
%. As explained previously, this low efficiency was probably related to the
nall shear and curvature in the isobars. The precipitation map, Fig. 42,

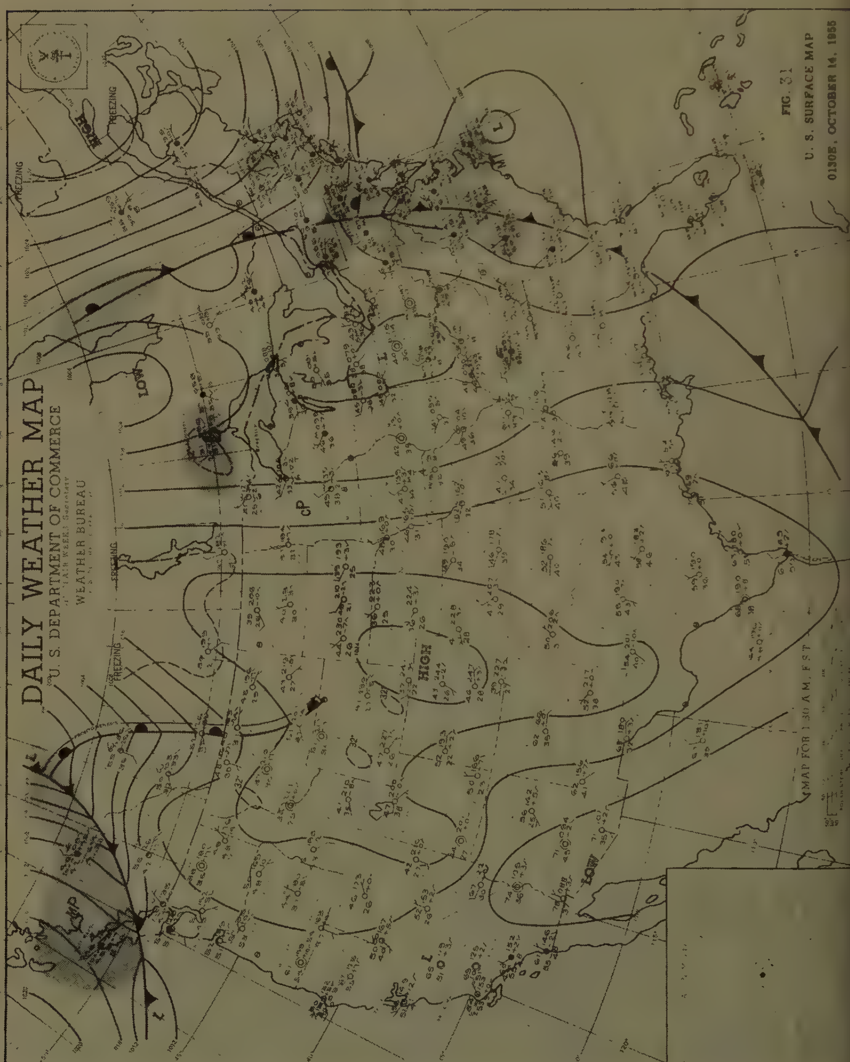

FIG. 31
U. S. SURFACE MAP
0130E, OCTOBER 14, 1955

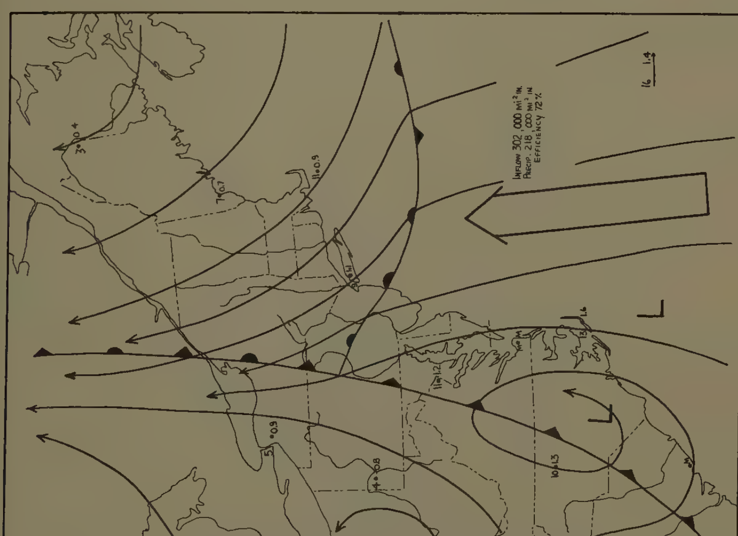

Fig. 32. Mean Trajectories of Air at 2000 Feet OCTOBER 14, 1955. See Fig. 7 for legend.

Fig. 33. Isohyetal Map, 24 Hrs. Ending 2400 EST OCTOBER 14, 1955. Precipitation in inches.

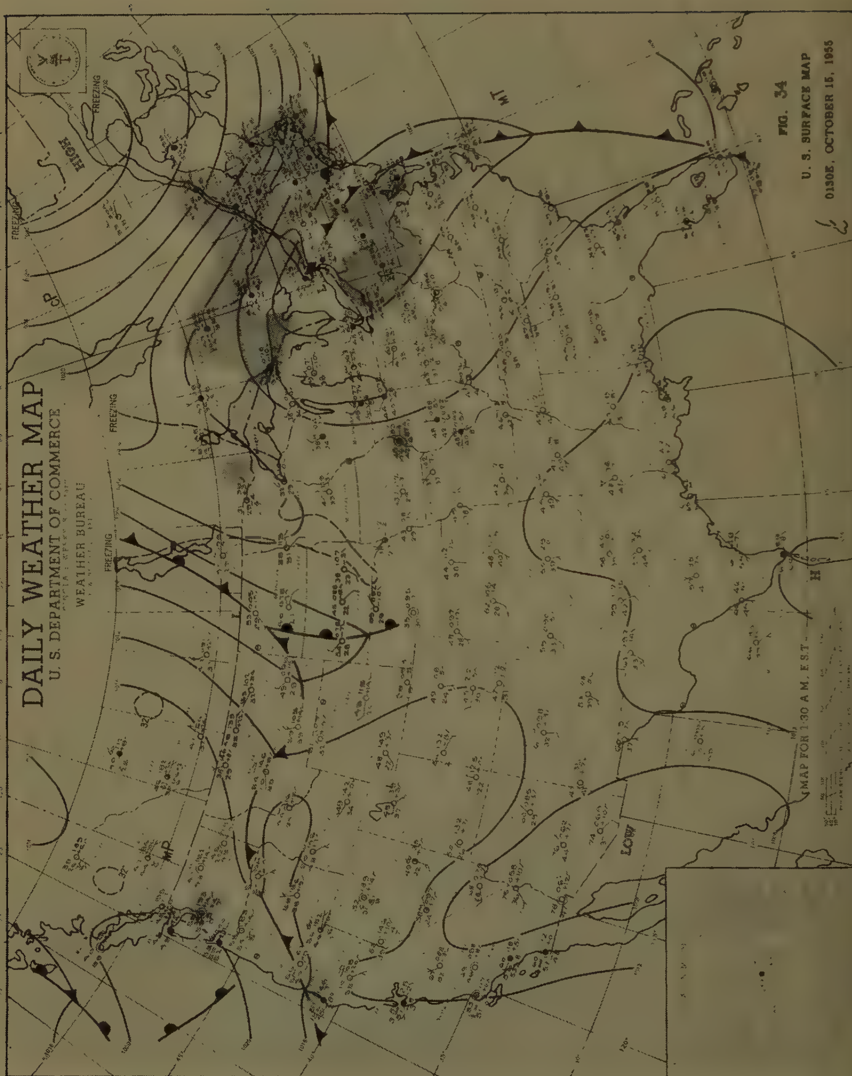

FIG. 34
U. S. SURFACE MAP
0130E, OCTOBER 15, 1955

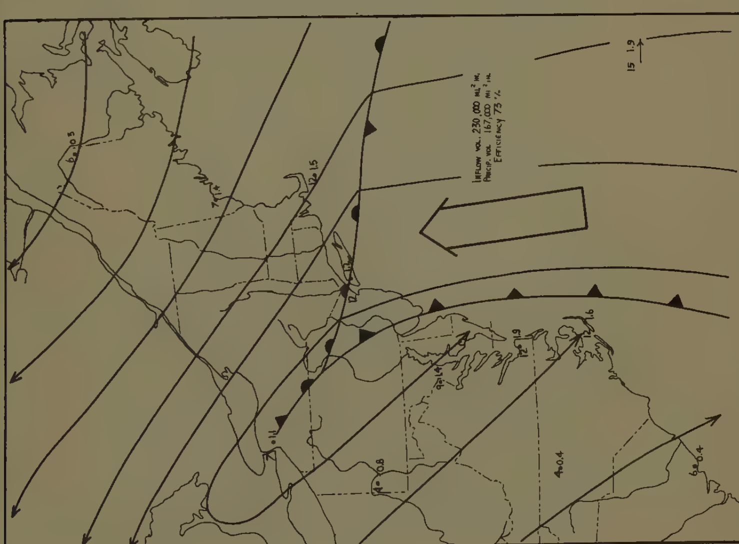

Fig. 35. Mean Trajectories of Air at 2000 Feet OCTOBER 15, 1955. See Fig. 7 for legend.

Fig. 36. Isohyetal Map, 24 Hrs. Ending 2400 EST OCTOBER 15, 1955. Precipitation in inches.

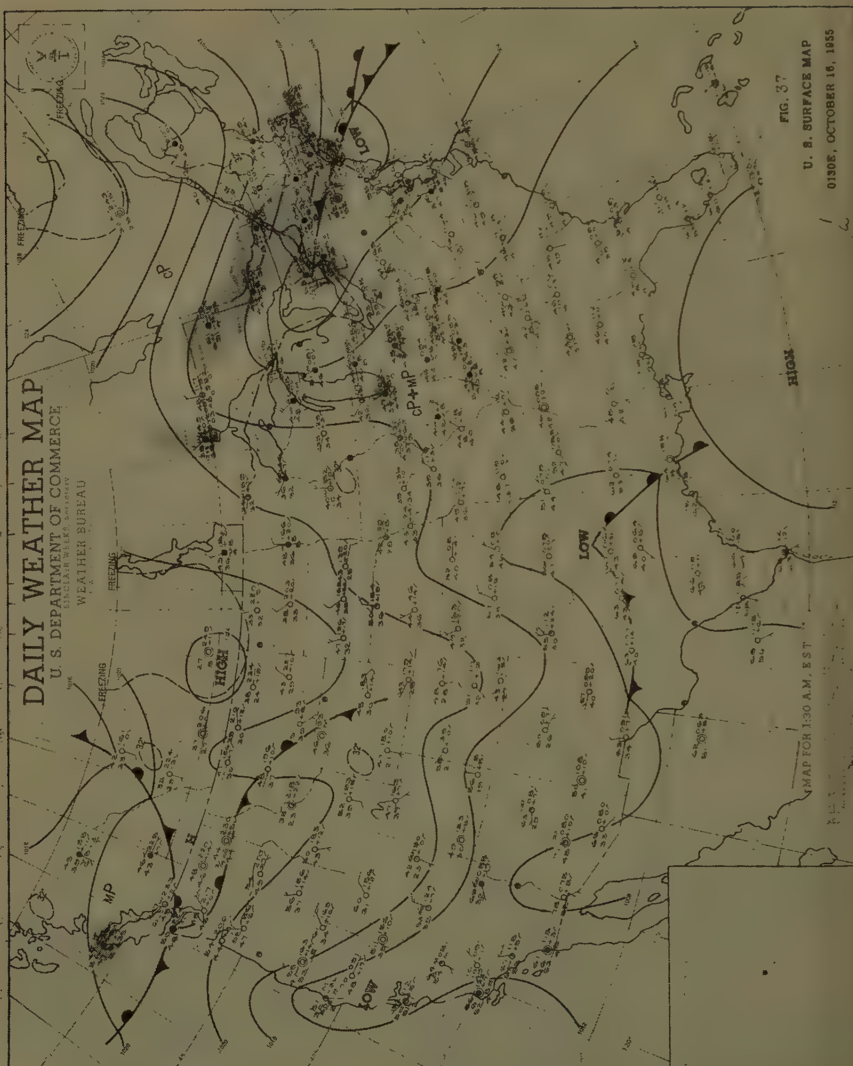

FIG. 37
U. S. SURFACE MAP
0130E, OCTOBER 16, 1955

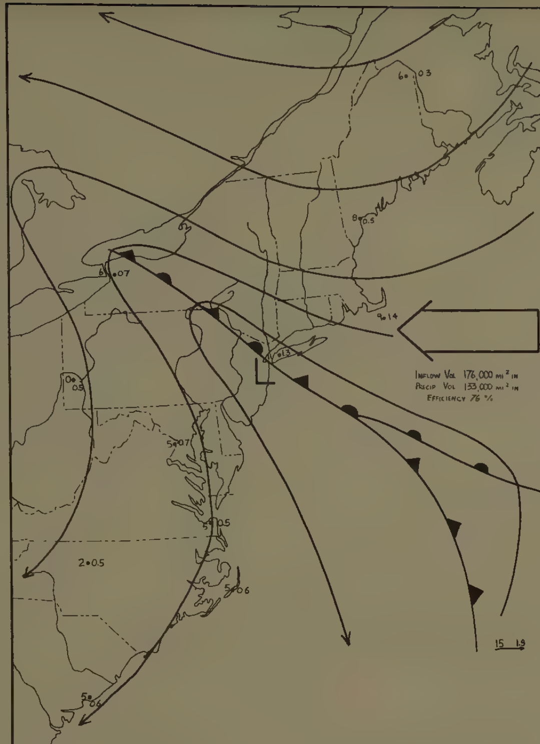

Fig. 38. Mean Trajectories of Air at 2000 Feet OCTOBER 16, 1955. See Fig. 7 for legend.

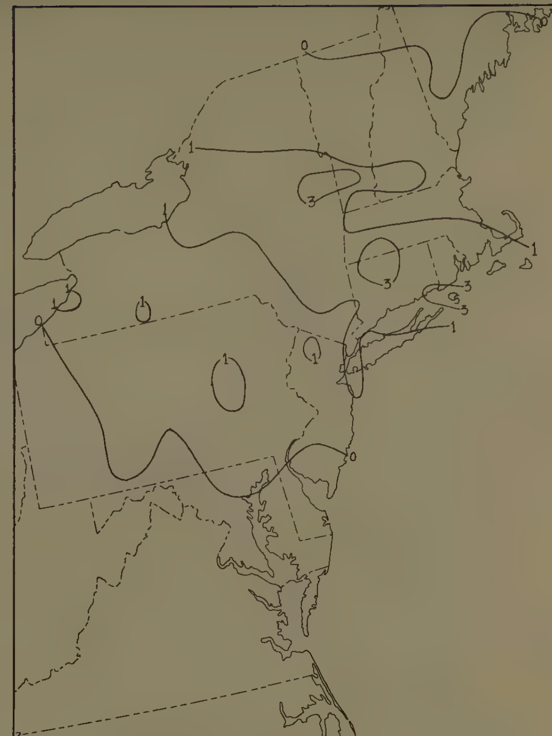

Fig. 39. Isohyetal Map, 24 Hrs. Ending 2400 EST, OCTOBER 16, 1955. Precipitation in inches.

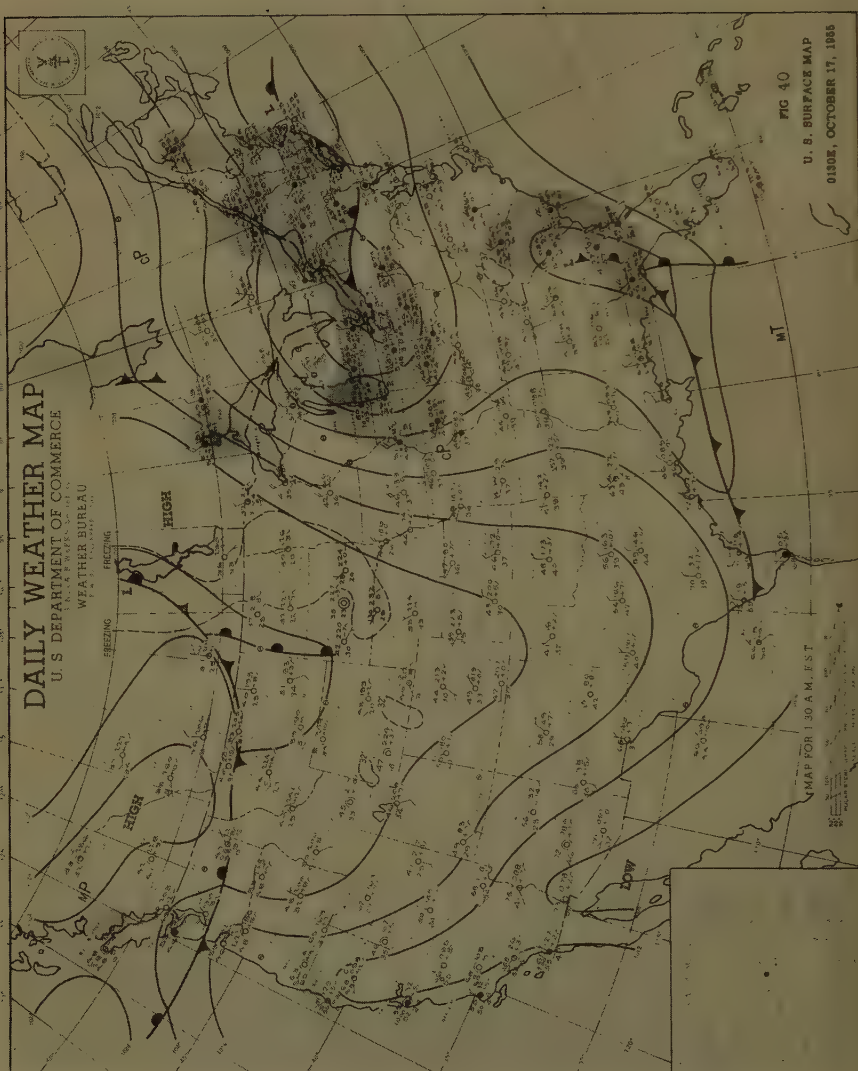

FIG 40
U. S. SURFACE MAP
0130E, OCTOBER 17, 1955

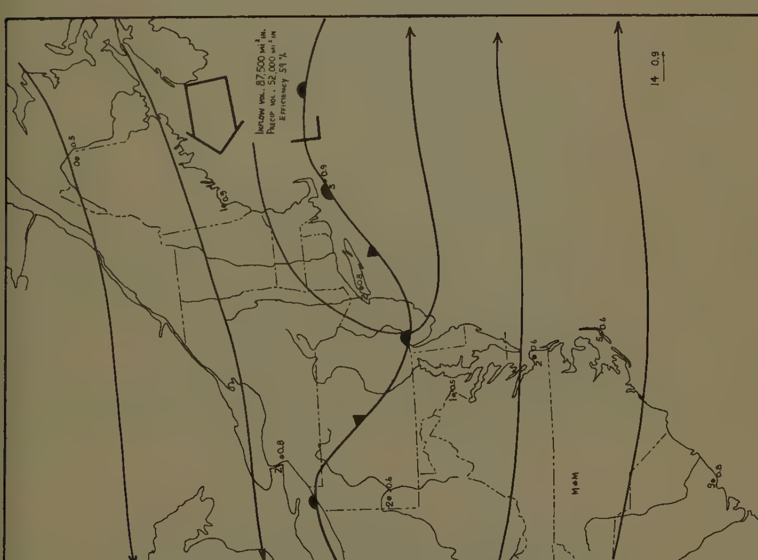

Fig. 41. Mean Trajectories of Air at 2000 Feet
OCTOBER 17, 1955. See Fig. 7 for legend.

Fig. 42. Isohyetal Map, 24 Hrs. Ending 2400 EST
OCTOBER 17, 1955. Precipitation in inches.

indicates three one-inch centers in central and northern New England. After the 17th the frontal system passed out to sea, the pressure circulation generally weakened and the precipitation occurring in the region ended. Fig. 43 is the total storm map for October 13-19.

SUMMARY

Hurricane Connie brought to its east side a large current of warm moist air which flowed northward into a region where the air already present was about 3 degrees centigrade colder from the surface to 10,000 feet than the air in the warm, moist current. A shallow trough formed in the region where these two air masses came into contact. Heavy precipitation began over Pennsylvania, New Jersey, New York, and part of New England even when the hurricane was 500-600 miles away. The air mass contrast, the trough of low pressure and the heavy precipitation continued as the storm with its accompanying moist, southerly current moved northward while cold air continued to come into the region from the north in approximately the first 10,000 feet and from the northwest from 10,000 feet to the top of the storm circulation.

The rainfall in the northeast in Diane was characterized by its concentration. This was associated with a narrow, concentrated current of warm air blowing into a region of pre-existing colder temperatures. As the hurricane entered the coast on August 17th a broad band of warm, moist air was blowing into the Middle Atlantic states. This air came into contact with air only 2 or 3 degrees centigrade cooler (up to 10,000 ft) so that the rainfall on this day was relatively light and widespread. The warm air current reached all the way to the Northeast, forming an air mass contrast and causing light rainfall over New England and eastern New York. On the 18th, the current of warm, moist air to the east of the hurricane had become much more concentrated and the air which this current was displacing was 5-6 degrees centigrade cooler. This situation resulted in a concentrated zone of rainfall over southern New England, southern New York and eastern Pennsylvania during the 18th and the morning of the 19th.

The October rainfall was caused by a stationary front which remained south of New England for almost 5 days. From October 13 to 15 this contrast was between warm, moist air coming from the south and cooler, drier air coming from the east. Both of these air masses met over the northeast where the air mass contrast between the surface and 10,000 ft amounted to 6 degrees centigrade. On October 16 and 17 the warm moist air current in the New England area became cut off and the air mass contrast was then due to the confluence of a cool current coming from the east and a colder current from the northeast which met over New England and New York. The mean surface-10,000 ft temperature difference between these currents was only 3 degrees centigrade, partly accounting for the lighter rainfall on these last two days of the storm.

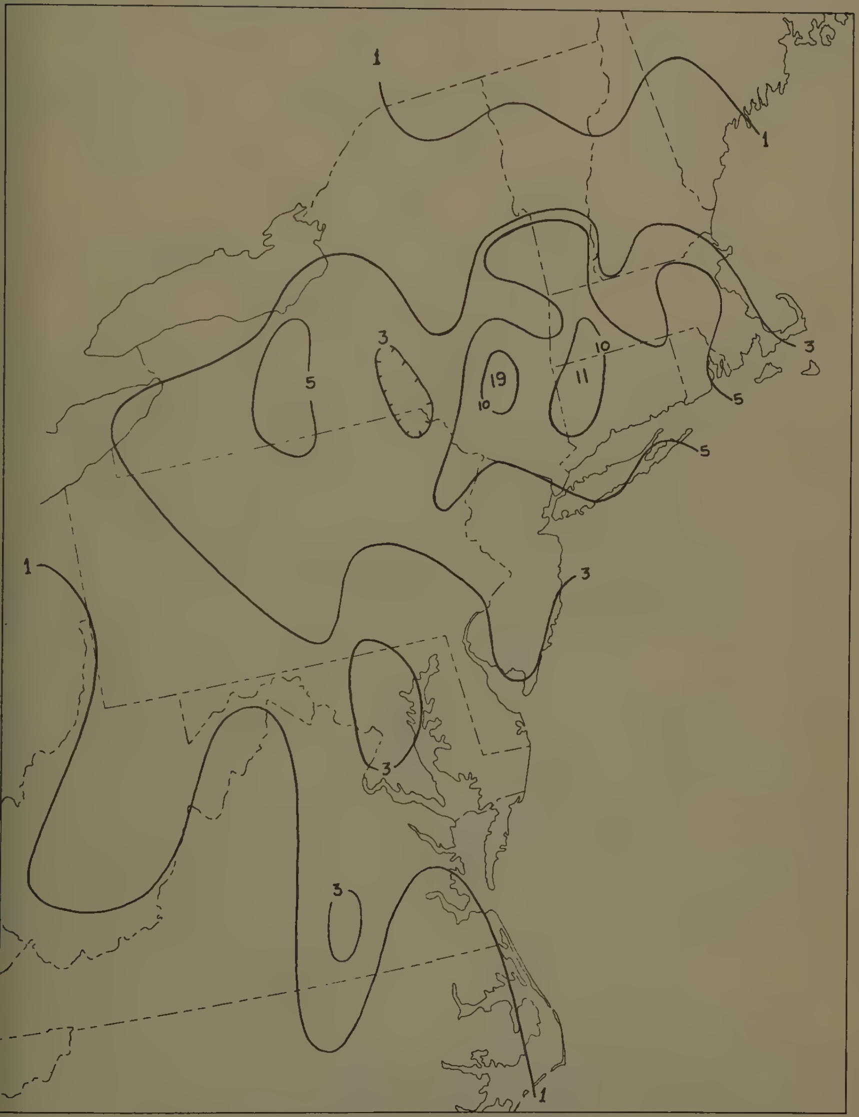

Fig. 43. Total Storm Isohyetal Map, OCTOBER 13-19, 1955.
Precipitation in inches.

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NORTHEASTERN FLOODS OF 1955: RAINFALL AND RUNOFF[a]

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(Proc. Paper 1662)

EXPLANATORY STATEMENT

The year 1955 produced a series of notable storms and floods that struck 1e northeastern states. Two of the storms were the results of hurricanes nd occurred in October. The symposium of three papers covers (1) the 1eteorological aspects of the storms, (2) the phenomenal discharges, and (3) 1e effect of the storms and floods on the hydrologic criteria used by the Corps f Engineers in the design of flood control structures.

The first paper concerning meteorology presents some of the physical easons for the occurrence of the rainstorms. The rain-producing and energy-roducing processes of hurricanes are described. Also considered are the nergy sources, the pressure distribution accompanying the release of rain, nd the volume of water vapor carried into the region by the moist, warm urrents.

The second paper briefly describes the floods of August and October 1955. utstanding peak discharges are listed for selected gaging stations, and a omparison made with the rainfall causing them. Also, a comparison is made ith past floods. Some indication of the frequency of the floods is presented.

The third paper describes the effect of the 1955 storms and floods on (1) ems pertaining to derivation of synthetic design floods, such as depth-area-uration rainfall relationships, unit hydrographs, and infiltration losses; (2) ood frequencies; (3) volume of runoff as it affects reservoir storage capacity nd regulation procedures; (4) method of transposing the storms to unaffected reas in New England; and (5) the design capacity of pumping stations for ocal protection projects.

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ote: Discussion open until November 1, 1958. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. Paper 1662 is part of the copyrighted Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers, Vol. 84, No. HY 3, June, 1958.

. Presented at meeting of Hydraulics Div., ASCE, Massachusetts Inst. of Technology, Cambridge, Mass., August, 1957.

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SYNOPSIS

This paper, one of three of a symposium, briefly describes the floods of August and October 1955. Outstanding peak discharges are listed for selected gaging stations, and a comparison made with the rainfall causing them. Also, a comparison is made with past floods. Some indication of the frequency of the floods is presented.

INTRODUCTION

The floods of August 1955, in the northeastern states, rank among the most destructive in the country's history. Record-breaking floods occurred in a broad region extending from eastern Massachusetts to southeastern Pennsylvania; this region is shown on map of Fig. 1. The floods were outstanding in: (1) the large geographic area covered by floods of such magnitude; (2) the extensive damage and loss of life, ranking with the greatest recorded in this country; (3) the degree to which prior records were exceeded; and (4) the fact that the greatest floods occurred predominately on the smaller streams.

Severe flooding occurred over an area of about 17,500 square miles. In addition, floods of less severity occurred in coastal states as far south as North Carolina.

A total dollar damage of $458 million has been reported. One hundred seventy-nine lives were lost, and 6,992 persons suffered major injury.

Peak discharges exceeded previously established maxima by 2.2 times on Blackstone River at Woonsocket, Rhode Island, by 4.1 times on Naugatuck River near Thomaston, Conn., and by 4.5 times on Bush Kill at Shoemakers, Pa. A unit discharge of 2,300 cubic feet per second per square mile came from the 2.5 square mile basin of Powdermill Brook near Westfield, Mass.

Floods were generally greatest on the smaller streams. However Connecticut River at Hartford reached the third highest stage since settlement, and the Delaware River between Port Jervis and Trenton exceeded the previous historic flood of 1903.

Severe flooding occurred again in October in an area centering about southwestern Connecticut and southeastern New York. Stages and discharges in the area exceeded those of August at many gaging stations.

Rainfall

The August floods resulted from excessive rainfall accompanying hurricane Diane. This rain fell upon ground wet from week earlier rains which accompanied hurricane Connie. The rains of August 11-16 amounted to from 9 to 11 inches in places. These were followed on August 17-20 by rains amounting to 17 to 19 inches at places. The largest rainfall occurred in a band along the southern third of Massachusetts and in northwestern Connecticut. Over 16 inches fell south of Boston, more than 17 inches near Worcester, and nearly 20 inches in the Westfield River basin west of Springfield. The distribution of the August 17-20 rains is shown on the isohyetal map presented as Fig. 2. The intensity of the rains may be judged from the mass rainfall curve for Hartford airport, shown as Fig. 3. Here there was

.43 inch of rain on August 17, 6.27 inches on the 18th, and 7.70 inches on the 9th, a total of 14.40 inches; practically all fell in the 1-1/2 days from 10 p.m. f the 17th to noon of the 19th. The total rainfall at selected gaging stations or the period August 17-20 is shown in Table 1.

The rainfall for the October storm centered over southwestern Massachusetts, western Connecticut, and southeastern New York. For the -day period October 14 to 16, the rainfall amounted to 14 inches in parts of Massachusetts and up to 15 inches in the Catskill Mountain area of New York. Analysis of this rainfall has not been made.

Runoff

The storm runoff, for period August 18-24, from a flood area of 25,700 square miles extending from Massachusetts to Pennsylvania, averaged slightly more than 3 inches (3.05). This average runoff was, of course, exceeded n many basins; the greatest runoff measured was 17.4 inches from the 60.6 square mile basin of Salmon Creek above Granby, Conn. The runoff for the August 18-24 period at selected gaging stations is shown in Table 1.

Discharge graphs for selected gaging stations, showing discharge for period August 10-25, are presented as Fig. 4. Also shown is the previously known peak discharge. It may be noted from these graphs that streams rose and receeded rapidly. These are typical of other streams in the area.

Rainfall-Runoff Relationship

The relation of total rainfall, for period August 17-20, to total runoff, for period August 18-24, at selected gaging stations is given in Table 1. The per cent of the rainfall that was measured as runoff also is shown in the table. These percentages vary from 31 to 115. The cause for one station showing more runoff than rainfall may be due to an unusually high base flow from a preceeding storm, or to inaccurate measurements of rainfall or runoff, or both.

The rainfall and runoff for gaging stations listed in Table 1 are shown plotted on Fig. 5. Also shown are the total yield and 53 per cent yield lines; 53 per cent yield is the average weighted by drainage area, for the stations listed.

Peak Discharge

Outstanding peak discharges for the August 1955 floods are listed in Table , and are plotted, discharge in cubic feet per second per square mile versus drainage area in square miles, in Fig. 6. A Myers curve has been fitted to the flood discharges. (The Myers curve is defined by the equation $Q = 100\,p\sqrt{DA}$, where Q is peak discharge in cfs, DA is drainage area in sq mi, and p is per cent of Myers scale or the "Myers rating".) A 50 per cent Myers curve, as shown on Fig. 6, seems applicable to this flood, although the highest floods show a rating of 70 per cent.

Outstanding peak discharges for the October 1955 floods are listed in Table 3 and are plotted in Fig. 7. A Myers curve fitted to the points in the same manner as for Fig. 6 shows a rating of 19 per cent.

Myers curves for the three largest floods in past years in the northeastern states were drawn for comparison. These were for the floods of November 1927, March 1936, and September 1938. A Myers curve was drawn for each flood, although the data do not exactly define a curve of this slope. The Myer ratings for these and for the August 1955 flood are:

August 1955,	50 per cent
September 1938,	36 per cent
November 1927,	30 per cent
March 1936,	27 per cent

These ratings are not a positive measure of the flood events, but they do show the relative magnitude of the four outstanding floods of recent times. Those curves are presented as Fig. 8.

Frequency of Floods

The frequency with which floods occur is an important element in almost every study of flood events. We speak of floods having frequencies of, say, 25, 50 or 100 years, as if these occur at regular intervals, so that if a 50-year flood occurs today it will be 50 years until the next one. This is not a correct interpretation, as a flood of any magnitude may occur at any time. More accurately, a 50-year flood is one that has a 2 per cent chance of occurring in any year; likewise a 100-year flood has a 1 per cent chance of occurring in any year.

For the great floods, few records are available and the correct evaluation of frequencies becomes practically impossible. Studies made of Connecticut streams indicate that the 1955 floods may be of a relatively short-term frequency or may be of a frequency well over 1,000 years. This is not a very satisfactory answer.

The Geological Survey has developed a procedure that provides a means for defining a flood-frequency curve at any point on any stream in a broad region, whether or not a gaging station is located at the site. This method provides two curves; one, the frequency curve, relating discharge, in ratio to the mean annual flood, to recurrence intervals, in years. The other curve relates the mean annual flood to size of drainage basin. The mean annual flood may be defined as the mean of all maximum annual flood peaks; for practical reasons, it is defined as the flood having a recurrence interval of 2.33 years.

Studies made of gaging station records from streams in the northeastern states, studies not yet completed, show a basic frequency curve similar to that presented in Fig. 9. This basic curve shows discharge ratios for the:

5 year flood as 1.4 times the mean annual flood
10 year flood as 2.0 times the mean annual flood
25 year flood as 3.7 times the mean annual flood
50 year flood as 5.8 times the mean annual flood
100 year flood as 9.0 times the mean annual flood

The mean annual flood has been computed from gaging station records for selected streams in the flood area; these are generally the streams experiencing the greater floods. A list of these stations, showing the maximum discharge and the ratio of this discharge to the mean annual flood, is shown in Table 4. A comparison of these ratios with the frequency curve of Fig. 9 may give some indication of the frequency of the 1955 floods.

A plot of discharge ratios vs drainage areas is presented as Fig. 10. This plot is not of great significance, but it does show that a few floods on the smaller streams were of greater frequency than those on larger streams. Shown on the figure is a curve that defines the upper limit of most of the data; this curve represents the magnitude of the highest floods experienced generally.

SUMMARY

Considered as a single event, the August 1955 flood was the greatest ex-
erienced since the area was settled by the white man. However, at certain
pecific locations greater floods have occurred in recent times.

The October 1955 flood produced higher peaks than the August 1955 flood
n streams in a relatively small area centering about southwestern
onnecticut and southeastern New York. In general, the October flood was
uch smaller than that experienced in August.

Based upon poor frequency definition, it may be said that the 1955 flood was
f a 100-year or larger recurrence interval; several streams experienced
uch greater than a 100-year recurrence interval flood.

Table 1.- Relation of rainfall, August 17-20, to runoff, August 18-24, 1955.

State and Stream	Drainage area (sq mi)	Rainfall (in.)	Runoff (in.)	Percent runoff
Massachusetts:				
Westfield River near Westfield. . .	497	16.6	5.1	31
Neponset River at Norwood	35.2	15.8	5.3	34
Quaboag River at West Brimfield . .	151	15.3	7.8	51
Little River at Buffumville	27.7	15.2	12.3	81
Chicopee River at Indian Orchard. .	688	14.4	4.6	32
Kettle Brook at Worcester	31.3	14.0	7.9	56
Rhode Island:				
Blackstone River at Woonsocket. . .	416	11.8	5.8	49
Connecticut:				
Salmon Brook near Granby	60.6	17.4	17.4	100
Burlington Brook near Burlington. .	4.12	15.4	8.2	53
Leadmine Brook near Thomaston . . .	24.0	14.8	8.5	57
Farmington River at Rainbow. . . .	584	14.7	10.1	69
Blackberry River at Canaan.	48.2	13.5	7.9	58
Scantic River at Broad Brook. . . .	98.4	13.5	6.3	47
New York:				
Wallkill River at Gardiner.	711	9.1	4.1	45
Fishkill Creek at Beacon.	186	8.2	3.5	43
Rondout Creek at Rosendale.	386	8.1	2.9	36
Neversink River at Godeffroy. . . .	302	8.1	3.8	47
New Jersey:				
Flat Brook near Flatbrookville. . .	65.1	7.7	5.4	70
Paulins Kill at Blairstown.	126	7.3	4.7	64
Pequannock River at Macopin Dam . .	63.7	6.8	3.4	50
Pennsylvania:				
Bush Kill at Shoemakers	117	10.2	7.0	69
Lehigh River at Stoddartsville. . .	91.7	9.4	10.8	115
Little Schuylkill River at Tamaqua.	42.9	9.0	4.7	52
Broadhead Creek at Minisink Hills .	259	8.8	8.5	97
Lackawaxen River at Hawley.	290	8.1	7.7	95
Weighted average.				53

Table 2.- Outstanding peak discharges, floods of August 1955.

State and Stream	Drainage area (sq mi)	Maximum discharge (cfs)	Cfs per sq mi
Massachusetts:			
Powdermill Brook near Westfield.	2.50	5,740	2,300
Lamberton Brook near West Brookfield	4.47	4,140	926
Stage Brook near Russel	5.21	4,910	942
Dickinson Brook near Granville	6.42	5,750	896
W.B. Farmington River near New Boston. . . .	92.0	34,300	373
Westfield River at Woronoco.	*189	61,500	325
Deerfield River near West Deerfield.	*374	43,700	117
Connecticut:			
Valley Brook near West Hartland.	7.20	8,260	1,150
W. B. Salmon Brook at West Granby.	11.7	10,500	897
E. B. Salmon Brook at North Granby	13.2	14,300	1,080
Salmon Brook near Granby	60.6	40,000	660
Naugatuck River near Thomaston	71.9	41,600	579
Still River at Robertsville.	84.4	44,000	522
W. B. Farmington River near Riverton	128	57,200	447
Naugatuck River near Waterbury	138	75,900	550
W. B. Farmington River at Riverton	216	101,000	468
Naugatuck River near Naugatuck	246	106,000	431
Farmington River near Collinsville	360	140,000	389
Housatonic River at Gaylordsville.	994	51,800	52.1
Connecticut River at Hartford.	10,480	210,000	20.0
New York:			
Bashbish Creek at Copake Falls	15.8	10,800	684
Delaware River near Barryville	*1,652	130,000	78.7
Delaware River at Port Jervis.	3,076	233,000	75.7
New Jersey:			
Delaware River at Montague	3,480	250,000	71.8
Delaware River at Belvidere.	4,535	273,000	60.3
Delaware River at Riegelsville	6,328	340,000	53.8
Delaware River at Trenton.	6,780	329,000	48.5
Pennsylvania:			
Wallenpaupack Creek at South Sterling. . . .	14.3	22,200	1,550
E. B. Wallenpaupack Creek at Greentown . . .	33.9	33,300	982
Pocono Creek near Stroudsburg.	37.7	22,400	594
Brodhead Creek at Analomink.	124	72,200	582
Brodhead Creek at Minisink Hills	259	68,800	266
Lehigh River at Walnutport..	889	77,800	87.5

* Area above reservoir not included.

Table 3.- Outstanding peak discharge, floods of October 1955.

State and Stream	Drainage area (sq mi)	Maximum discharge (cfs)	Cfs per sq mi
Massachusetts:			
North River at Shattuckville	88.4	13,200	149
North Nashua River near Leominister. . . .	107	8,870	82.9
Deerfield River at Charlemont.	*178	18,100	102
Deerfield River near West Deerfield. . . .	*374	43,700	117
Connecticut:			
Fivemile River near New Canaan	5.85	2,140	366
Norwalk River near Branchville	7.54	3,100	411
Pequonock River near Trumbull.	14.4	4,110	285
Still River at Danbury	38.3	5,000	131
Still River at Lanesville.	68.5	7,980	116
Saugatuck River near Westport.	77.5	14,800	191
Salmon River near East Hampton	105	9,130	87.0
Housatonic River at Stevenson.	1,545	75,800	49.1
New York:			
Burheight Creek near Spencer	1.35	597	442
Beechy Bottom Creek near Bear Mountain . .	8.68	2,010	231
Kaaterskill Creek at Palenville.	14.2	4,820	340
Chestnut Creek at Grahamsville	20.9	4,640	222
E. B. Croton River near Brewster	79.5	8,130	102
Nanticoke Creek at Union Center.	89.7	9,900	110
Normans Kill near Slingerlands	169	13,300	78.8
Schoharie Creek at Prattsville	236	55,200	234
Croton River at New Croton Dam	378	45,400	120
Rondout Creek at Rosendale	386	35,800	92.8
Esopus Creek at Saugerties	425	23,900	56.3
Wallkill River at Gardiner	711	30,800	43.3
Schoharie Creek at Burtonsville.	883	76,500	86.7
Chemung River at Chemung	2,530	89,000	35.1
Mohawk River at Cohoes	3,456	100,000	28.9
Susquehanna River near Waverly	4,780	72,100	15.1
Hudson River at Green Island	8,090	113,000	14.0
New Jersey:			
Ramapo River near Mahwah	118	10,900	92.4
Ramapo River at Pompton Lakes.	160	12,000	75.0
Pennsylvania:			
Cory Creek near Mainesburg	12.2	2,210	181

* 184 sq mi above Harriman Reservoir not included.

Table 4.- Ratio of peak discharge of August 1955 flood to mean annual flood.

State and Stream	Drainage area (sq mi)	Maximum discharge (cfs)	Ratio to mean annual flood
Massachusetts:			
Little River at Buffumville	27.7	8,340	28.5
Quinebaug River at Westville.	93.8	17,500	22.6
W. B. Farmington River near New Boston. . .	92.0	34,300	14.5
Quaboag River at West Brimfield	151	12,800	11.1
Blackstone River at Northbridge	139	16,900	10.9
Kettle Brook at Worcester	31.3	3,970	7.9
Mumford River at East Douglas	27.8	2,140	6.7
Chicopee River at Indian Orchard.	688	40,500	6.5
W. B. Westfield River at Huntington	93.7	26,100	6.3
Mill River at Springfield	33.9	1,960	6.1
Rhode Island:			
Blackstone River at Woonsocket.	416	32,900	5.6
Connecticut:			
Quinebaug River at Quinebaug.	157	49,300	25.9
Shepaug River near Roxbury.	133	50,300	14.8
W. B. Farmington River at Riverton.	216	101,000	13.5
Scantic River at Broad Brook.	98.4	13,300	12.1
Quinebaug River at Putnam	331	48,000	12.0
Naugatuck River near Naugatuck.	246	106,000	11.8
Naugatuck River near Thomaston.	71.9	41,600	11.6
Willimantic River near South Coventry . . .	121	24,200	11.5
Pomperaug River at Southbury.	75.3	29,400	10.3
Farmington River at Rainbow	584	69,200	8.6
Shepaug River at Woodville.	38.0	13,800	8.6
Pennsylvania:			
Bush Kill at Shoemakers	117	23,400	10.6
Lehigh River at Stoddartsville.	91.7	31,900	10.1
Lehigh River at Tannery	322	58,300	8.1
Little Schuylkill River at Tamaqua.	42.9	7,790	5.2

Figure 1. - Map of area of severe flooding, August 1955.

Figure 2. - Map of flood area, showing total rainfall, August 17-20, 1955.

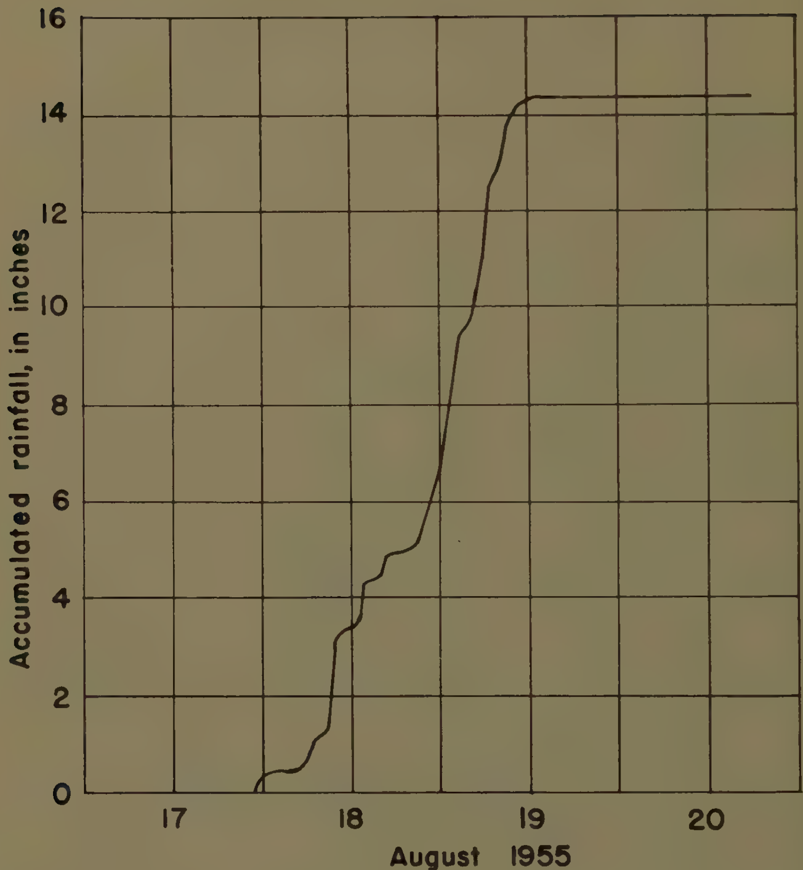

Figure 3. - Mass rainfall curve for Hartford airport.

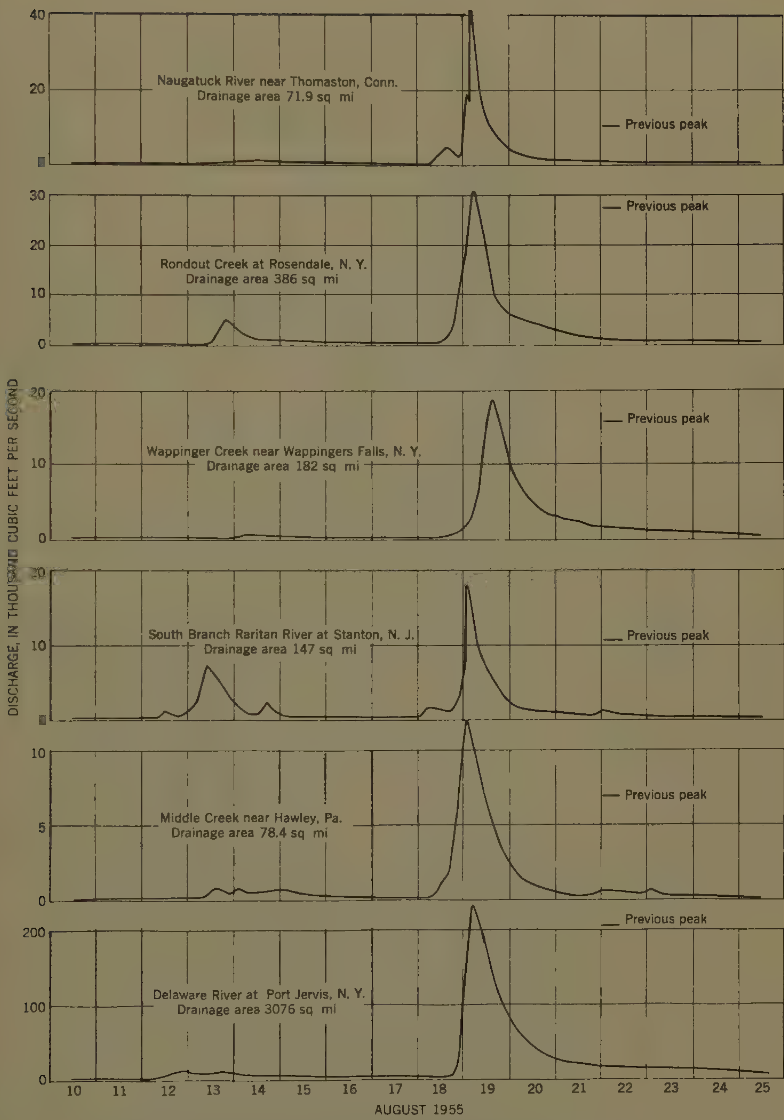

Figure 4. - Graphs of discharge at selected gaging stations.

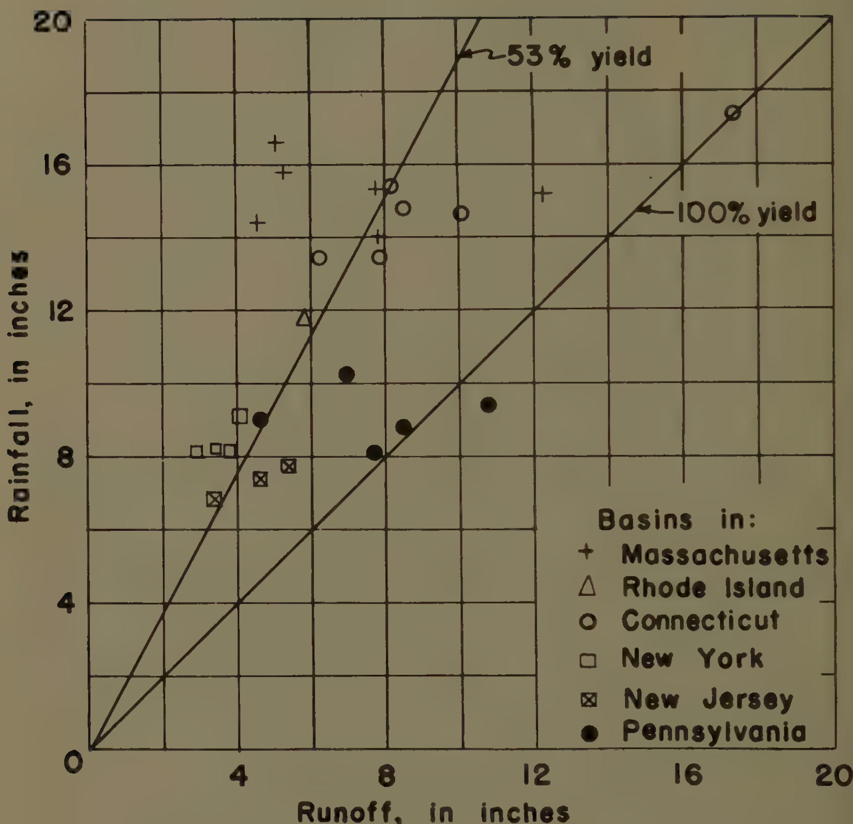

Figure 5. - Relation of rainfall, August 17-20, to runoff, August 18-24, 1955.

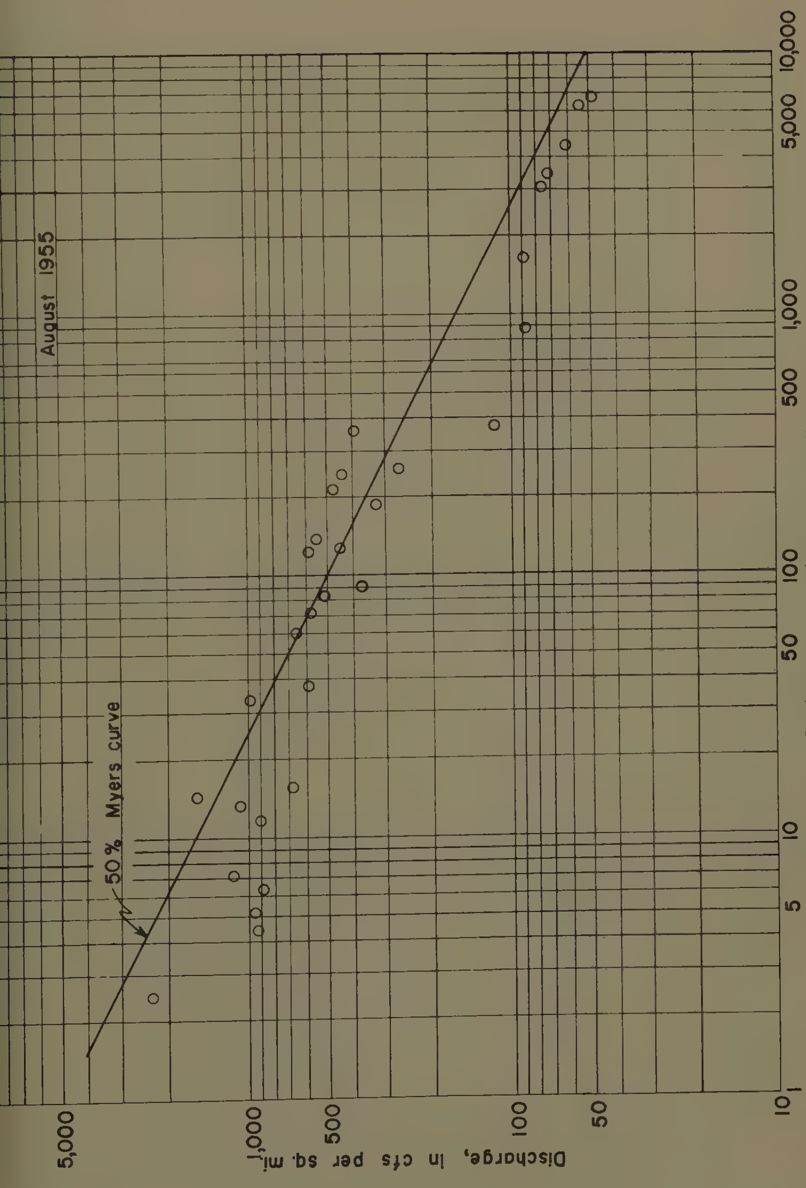

Figure 6. - Relation of peak discharge to drainage area, August floods.

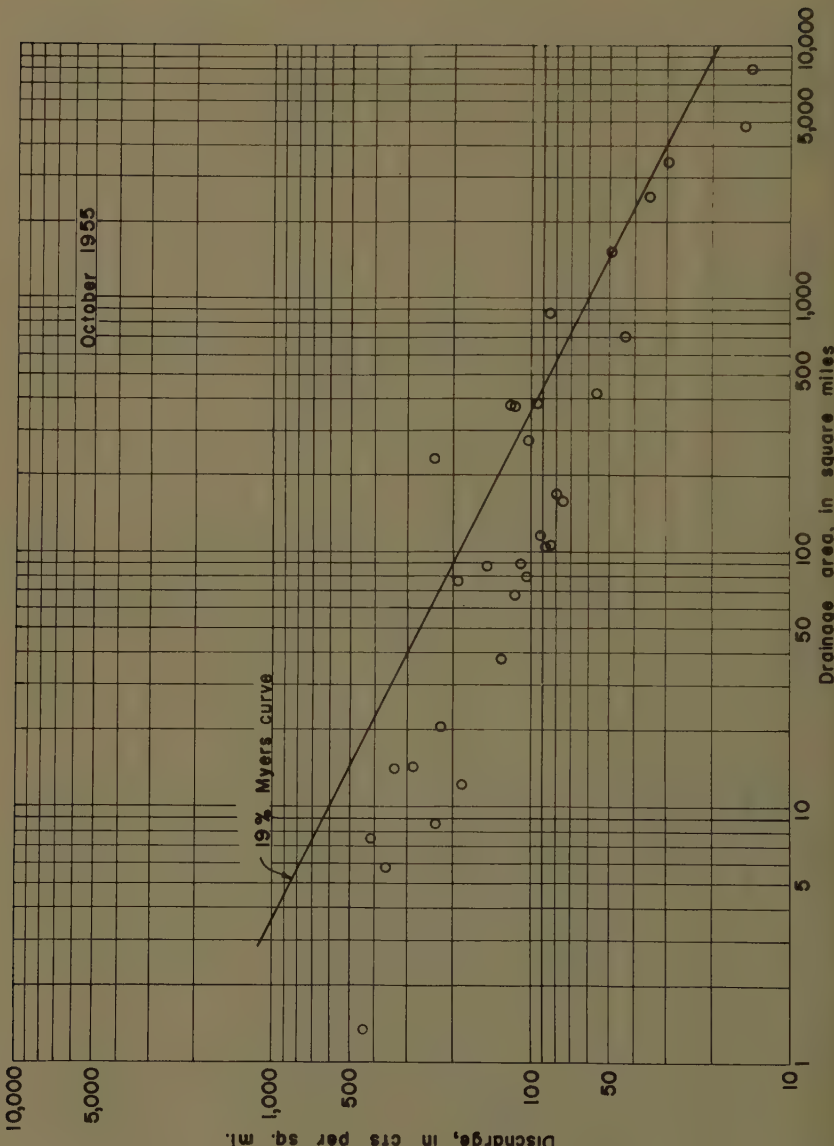

Figure 7. - Relation of peak discharge to drainage area, October floods.

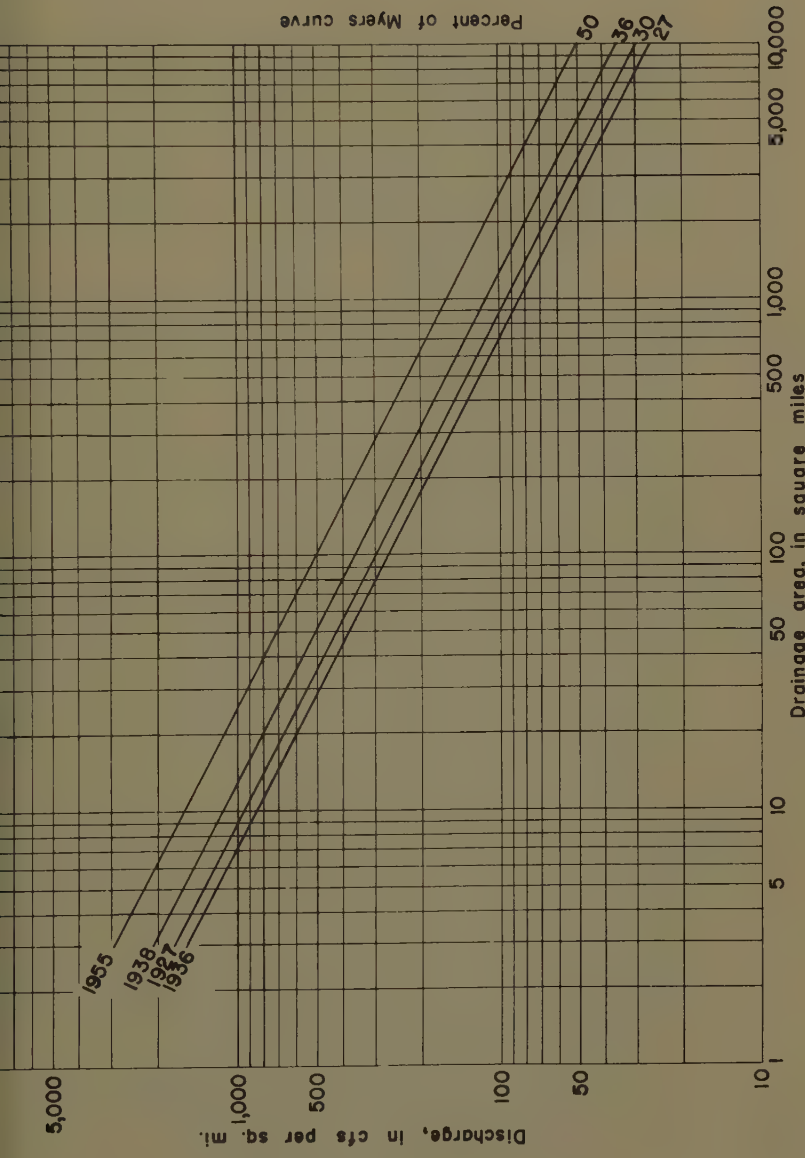

Figure 8. - Relative magnitude of outstanding floods.

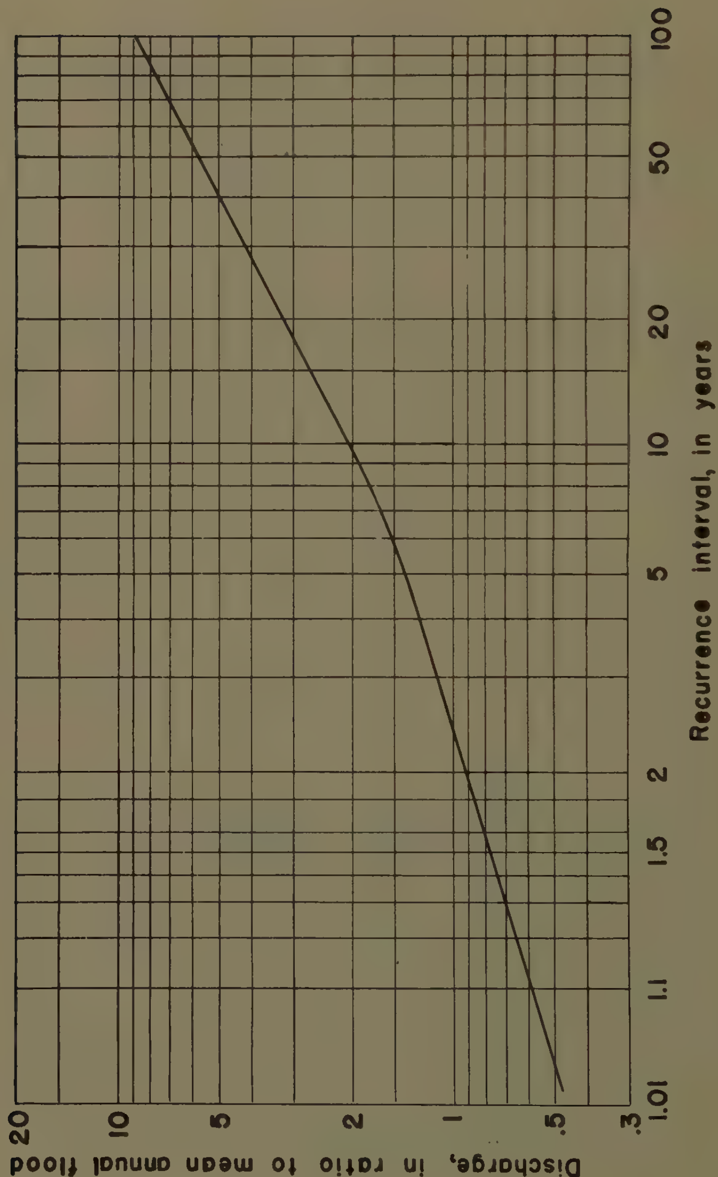

Figure 9. - Basic flood-frequency curve.

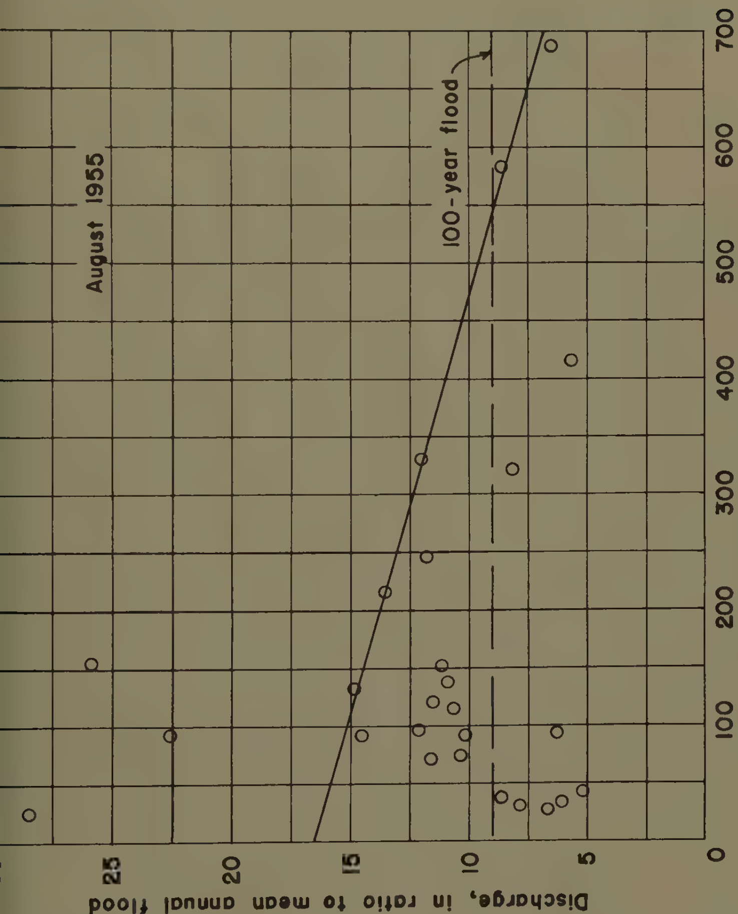

Figure 10. - Ratio of peak discharge to mean annual flood.

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NORTHEASTERN FLOODS OF 1955: FLOOD CONTROL HYDROLOGY[a]

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(Proc. Paper 1663)

EXPLANATORY STATEMENT

The year 1955 produced a series of notable storms and floods that struck northeastern states. Two of the storms were the results of hurricanes l occurred in October. The symposium of three papers covers (1) the teorological aspects of the storms, (2) the phenomenal discharges, and (3) effect of the storms and floods on the hydrologic criteria used by the Corps Engineers in the design of flood control structures.

The first paper concerning meteorology presents some of the physical isons for the occurrence of the rainstorms. The rain-producing and energy-ducing processes of hurricanes are described. Also considered are the ergy sources, the pressure distribution accompanying the release of rain, l the volume of water vapor carried into the region by the moist, warm rents.

The second paper briefly describes the floods of August and October 1955. standing peak discharges are listed for selected gaging stations, and a nparison made with the rainfall causing them. Also, a comparison is made h past floods. Some indication of the frequency of the floods is presented.

The third paper describes the effect of the 1955 storms and floods on (1) ms pertaining to derivation of synthetic design floods, such as depth-area-ation rainfall relationships, unit hydrographs, and infiltration losses; (2) od frequencies; (3) volume of runoff as it affects reservoir storage capacity l regulation procedures; (4) method of transposing the storms to unaffected as in New England; and (5) the design capacity of pumping stations for local tection projects.

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e: Discussion open until November 1, 1958. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. Paper 1663 is part of the copyrighted Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers, Vol. 84, No. HY 3, June, 1958.

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SYNOPSIS

The notable storms and floods of August and October 1955 resulted in a re-appraisal of the hydrologic criteria used in the design of flood control structures. Items investigated are depth-area-duration rainfall relationships, unit hydrographs, flood frequencies, volume of runoff, pumping capacity for local protection projects, and a method of storm transposition. Further study is necessary to properly evaluate some of the hydrologic phenomena produced by the record floods.

INTRODUCTION

On the basis of the storms and floods of 1955, it has been necessary to re-appraise the hydrologic criteria developed by the Corps of Engineers for the design of flood control projects in the northeast states, and to determine whether these criteria should be revised in order to insure adequate flood control measures. The principal subjects requiring re-analyses are: (1) items pertaining to derivation of synthetic design floods, such as depth-area-duration rainfall relationships, unit hydrographs, and infiltration losses; (2) flood frequencies; (3) volume of runoff as it affects reservoir storage capacity and regulation procedures; (4) method of transposing the "Diane"-storm to unaffected areas; and (5) the design capacity of pumping stations for local protection projects.

It will not be possible to cover these subjects thoroughly in this paper; first, because of the complexities of the subjects and the length required to describe each topic, and secondly, because satisfactory solutions and explanations have not been evolved for some of the complex phenomena. In some instances, the effect of the 1955 storms and floods on specific hydrologic criteria will be discussed; in others the effect is still a matter of conjecture, and further deliberations and counsel will be required to reach adequate and satisfactory explanations.

Most basic data concerning rainfall and flood discharges are obtained from records and publications of the U. S. Weather Bureau and the U. S. Geological Survey. Interpretation and analysis of these data, as described in this paper, conform to the general practice of the Corps of Engineers in developing criteria for flood control planning and design. Comments on various items, such as unit hydrographs and flood frequencies, are the writer's personal speculations and do not necessarily have the concurrence of the Corps of Engineers.

Spillway Design Floods

During the past 20 years the Corps of Engineers has developed a rather standard procedure for the determination of design floods as criteria for determining the length and design surcharge of spillways for major dam projects. A prime consideration in the design of major flood control dams is the provision for safety against failure by great floods. Hence, the dams, which in general consists of high earth dikes capable of impounding large volumes of water, and some type of spillway, are designed for a combination of critical meteorological conditions. The spillway design flood is developed

from three basic elements; (1) precipitation, (2) runoff losses, and (3) the unit hydrograph which represents the relation between rainfall and runoff. It is necessary to consider each of these elements separately and compare them with the experiences of 1955.

Precipitation

The storms of 1955 produced not only great amounts of precipitation but also high intensities during short periods of time as shown in Table 1. These are not record amounts for isolated storms concentrating over relatively small areas, but are noteworthy because they represent wide areal coverages. Table 2 shows depth-area-duration relationships for the storms of August 11-15, 1955 ("Connie") and August 17-20, 1955 ("Diane"). (Similar data for the storm of October 14-17, 1955 have not been determined.) Also shown in Table 2 are the rainfall amounts for the probable maximum precipitation and the standard project storm. Both are synthetic storms in common use by the Corps of Engineers for the design of flood control structures.

The probable maximum precipitation is used as a basis for deriving the spillway design flood for high dams where it is considered mandatory to insure that the spillway is adequate to prevent overtopping of the structure during the most severe combination of meteorological and hydrological conditions considered reasonably possible. It is desirable to note however that not all dams are designed against the maximum possible flood. When possible failure of a dam from overtopping during extreme floods would not result in serious danger to life or extraordinary property damage downstream, a policy less severe is prescribed.

It will be noted in Table 2 that the probable maximum precipitation exceeds the depths of rain experienced in the August 17-20 storm for all durations but the differences are greater for shorter durations. For 200 square miles, and for durations of 6 and 12 hours, the probable maximum precipitation exceeds the 1955 storm by 105 and 78 per cents. For 24 and 48 hour durations, the exceedence drops to 44 and 20 per cents. Comparable figures for a drainage area of 1,000 square miles, and for durations of 6, 12, 24 and 48 hours, are 73, 49, 29, and 7 per cents. These percentages clearly show the phenomenal magnitude of the August 17-20 storm. Rainfall depth-duration curves are shown on Plate 1 for the August 1955 storm center at Westfield, Massachusetts as compared with the probable maximum precipitation and the standard project storm.

For watersheds less than 500 square miles no change in the probable maximum precipitation, as used to determine spillway design floods, is presently contemplated as a result of the August storms. Studies have demonstrated that in the northeastern states the rivers on which flood control projects are located are generally flashy and have a relatively short period of concentration. On such streams the short periods of high rainfall intensity produce the high peak discharges that ordinarily provide the design criteria for the spillway length and surcharge. Most projects control drainage areas less than 500 square miles, and for these watersheds the probable maximum precipitation substantially exceeds the August ("Diane") storm.

For a project controlling a watershed of 500 square miles or more, and where the runoff characteristics show that storms of long duration produce critical conditions for spillway design, it will be advisable to review the design storm values. Such a review should be made by experienced

meteorologists to re-evaluate the probable maximum precipitation applicable for that specific area.

Infiltration and Other Losses

The floods in August and October 1955 substantiate the fact that flood peaks and volumes of runoff are greatly influenced by antecedent conditions. The storm of August 11-14, 1955 ("Connie") followed a very dry period in the northeastern states. The ground was parched, and the rivers and lake levels were below normal. The observed runoff from this storm was low, and the peak discharges were generally well below flood stages. Infiltration, storage, and other losses were very high.

The "Diane" storm of August 17-20, 1955, following on the heels of "Connie" and, covering about the same area, provides another example in which extraordinary storms and floods have occurred in relatively close sequence. The rainfall on the 17th was generally light and the runoff was insignificant. The precipitation on the 18th was heavy and in the western sections of New England occurred principally between 6 a.m. and 4 p.m. This rain thoroughly soaked the ground and produced spillway discharges from many ponds and lakes. Rivers exceeded their bankfull capacity and minor damage was experienced in many locations. The heaviest rain occurred between 9 p.m. on the 18th and 9 a.m. on the 19th. There was now no place for the water to go except into the river valleys. Infiltration, surface detention, and other losses were insignificant and almost the entire rainfall, augmented by the water released from numerous ponds and lakes by dam failures, went into the surface runoff and flowed into the already overtaxed rivers.

Analyses of the flood hydrographs were made in an attempt to determine the amount of the losses. Due to the difficulties in estimating average rainfall amounts over watersheds, and the uncertainties in the volume of flood runoff, it was not possible to make any precise determination. However it was found from investigating 46 different hydrographs that the apparent rate of infiltration and other losses varied between a minimum of 0.04 and a maximum of 0.25 inches per hour. The overall average approximated 0.1 inch per hour.

It has been the practice of the Corps of Engineers in New England to assume a loss of 0.05 inches per hour during the probable maximum precipitation in order to determine the rainfall excess, or that part of the storm that becomes surface runoff. The experiences of August 1955 substantiate this assumed low rate, and even indicate that during the high intensities of the spillway design storm, infiltration and other losses could be entirely neglected, especially when it is assumed there had been wet antecedent conditions.

Unit Hydrograph

The unit hydrograph is a very important factor in deriving the spillway design flood, particularly for projects where the peak of the flood, rather than the volume, has the greatest influence on determining the length and head on the spillway. Studies are first made of all flood hydrographs at the proposed dam-site, or at gaging stations where the flood runoff characteristics are comparable, in order to select a unit hydrograph for that particular project. All applicable flood and rainfall records are investigated to determine the correlation between the storm and the flood. These various unit hydrographs are then compared for similarity, considering the relative magnitude of the storms and floods. Such studies frequently show that the peak and time of

concentration of a unit hydrograph will vary with the magnitude of the flood,—the larger the flood, the higher the peak of the unit hydrograph, and the shorter the time of concentration.

Plate 2 illustrates the variations in the unit hydrographs derived from floods of record for the gaging station on the Naugatuck River at Thomaston, Connecticut. The peak of the unit hydrograph derived from the 1955 flood is more than double the peak derived from a minor flood, and is 50 to 80 per cent in excess of the unit hydrographs computed from the previous record floods in 1938 and 1948.

The derived unit hydrograph with the highest peak and shortest time of concentration is usually selected as the basic unit hydrograph. In the development of the spillway design flood, the peak of this graph is frequently increased some arbitrary percentage and the time of concentration shortened in order to simulate the rainfall-runoff relationships that might be anticipated during the probable maximum precipitation.

It is a well known fact that on streams with steep slopes having little valley storage, channel hydraulics is an important factor in influencing the magnitude of the peak discharge. These same hydraulic characteristics are reflected to a considerable degree in the unit hydrograph. Hence the unit hydrograph, a hydrologic function ordinarily considered the relating factor between rainfall and runoff, is also utilized to evaluate the channel hydraulics. The variations in the unit hydrographs for floods of different magnitude as shown in Plate 2 result from this application.

Initial runoff, following a storm, occurs as overland flow. Upon entering a defined watercourse the runoff becomes channelized. The character of the flow changes as the runoff progresses from overland to channelized flow and quantitative separation of the total flow into these two components, which vary widely during the course of a storm, is difficult and usually is not attempted.

The relationships between rainfall and both overland and channelized flow undoubtedly vary with the intensity and amount of rainfall. It is conjectured, however, that the variations are much greater for channelized runoff than for overland flow. Major variations are considered to result primarily from alterations in the channel hydraulics, which in some rivers undergo pronounced changes during major floods.

The Naugatuck River in Connecticut is typical of this type of river. The Naugatuck River maintains a nearly uniform drop of 14 feet per mile for the entire distance of 40 miles from Torrington, Connecticut to its confluence with the Housatonic River at Derby. The river runs in a narrow valley where the channel and over-bank storage is relatively small and has very little effect on modifying the flood peaks.

During the "Diane" flood the computed peak discharge at the river gage at Thomaston was 41,600 c.f.s., which is equivalent to a rate of runoff of 0.9 inches per hour from the drainage area of 72 square miles. The rate of precipitation on the watershed, however, probably did not greatly exceed a basin-wide average of 1 inch per hour. Rates of runoff almost equal to the rate of rainfall would normally be expected only from the proverbial "tin-roof" and not from 72 square miles of watershed, a large portion of which is forested. Several small dams, located some distance upstream, failed during this flood, but their volume of release was insignificant compared with the total volume of the flood.

A similar relationship occurred further downstream on the Naugatuck River in the Town of Naugatuck. The discharge here, for a drainage area of

246 square miles, was 106,000 c.f.s., equivalent to a rate of runoff of nearly 0.7 inches per hour over the entire watershed. It is inconceivable that this high rate of runoff is directly related to the rate of precipitation.

It is suspected that hydraulic conditions within the Naugatuck River and other rivers with similar characteristics, change radically during a major flood. The hydraulic radii increase, thus increasing the channel conveyance. The hydraulic gradient changes from a series of short variable slopes to the overall slope of the river valley, and due to the rapid rise in the river stages the hydraulic gradient at the front of the flood wave is probably even steeper than the valley slope. As the flood velocities increase, the stream bed in alluvial areas is scoured which often increases the effective cross-sectional water area during the flood crest.

It is believed that the combination of these complex phenomena contributed to the high flood peaks experienced on the many flashy streams during the August and October floods. These factors very likely produce a pyramiding effect on the flood crest, and may even tend to change the flow characteristics from sub-critical to near critical, and in some steep streams to super-critical conditions. These changes in the discharge characteristics accelerate the velocity of flow which may form a wave or a surge as the crest speeds downstream to the mouth. Temporary stoppages at debris-choked bridges and natural restrictions, such as occurred on the Naugatuck River, cause secondary surges when the bridges finally fail from the swollen flood stages.

In conclusion, it is the writer's belief that the unit hydrograph, now used as the link between rainfall excess and a point of discharge in a river should be divided into two relationships: (1) the unit hydrograph to represent the relation between the rainfall excess and overland runoff, and (2) flood conveyance factors (or some other terminology) to evaluate the changes in channel hydraulics during large flood flows that will be reflected in either direct hydraulic computations or by the flood routing coefficients. However, until such procedures are derived it will be necessary to utilize the unit hydrograph to represent the entire relationship between rainfall excess and runoff. Considerable judgment will be required to increase the peaks of unit hydrographs, when determined from relatively minor floods, to compensate for the uncertain flow conditions that will exist during a storm comparable to the probable maximum precipitation. Such judgment must be applied by experienced hydrologists to properly allow for the many factors and variables influencing flood discharges.

Summary

The following summarize the effect of the 1955 storms and floods on the derivation of the spillway design flood:

1. The probable maximum precipitation for the northeastern states, as evolved by the U. S. Weather Bureau prior to 1955, is adequate for developing the design flood for watersheds generally up to 500 square miles, but should be reviewed by meteorologists for larger basins, and for those particular areas where extended durations of rain have a significant effect on the design criteria.
2. Infiltration and other losses during a severe storm following wet antecedent conditions probably do not exceed 0.05 inches per hour and for practical purposes may be entirely neglected.

3. The unit hydrograph has inherent weaknesses in its application to storms of various magnitude and intensities, but until the flood hydraulics of rivers have been further developed, it remains the best tool for relating the rainfall and runoff.

Standard Project Flood

The standard project flood is a synthetic flood used by the Corps of Engineers to measure the flood potentialities of a river basin. It represents flood discharges that may be expected from the most severe combination of meteorological and hydrologic conditions that are considered reasonably characteristic of the geographical region involved, excluding extremely rare combinations. The standard project flood is used as criteria for establishing design grades for walls and dikes in local protection projects, determining the design capacity of channel improvements, and checking the effectiveness of flood control reservoirs.

The standard project flood is derived from generalized precipitation curves prepared by the Corps of Engineers. The rainfall amounts and intensities are approximately one-half the probable maximum amounts and rates, as determined by the Hydrometeorological Section of the Weather Bureau, and shown on Plate 1 for an area of 200 square miles in the vicinity of Westfield, Massachusetts. Up to six hours duration, the rainfall rates for the standard project storm exceed the maximum recorded rates for the "Diane"-storm. For longer durations, however, the "Diane"-storm exceeds the standard project flood; for 24 hours duration, for example, the exceedence is 47 per cent.

In areas most severely affected by the "Diane"-storm the experienced flood exceeded the standard project floods computed prior to 1955. For example, the August 1955 floods in the Naugatuck and upper Quinebaug Rivers exceeded the previous standard project floods by 30 and 15 per cent respectively. The August 1955 flood also exceeded the standard project floods on the tributaries in the lower Connecticut River Basin.

Many of the tributary river basins in the northeast states are small, and major floods usually develop from high-intensity short-duration rainfall. For these rivers the standard project storm, without modification, will produce flood peaks approximating or slightly higher than the "Diane"-flood, if it is assumed that there had been wet antecedent conditions to thoroughly saturate the ground and to fill natural storage basins and the river channels. High unit hydrographs with short times of concentration, comparable to those developed from analyzing the "Diane"-flood, are applicable for these conditions and result in high peak discharges. Thus the principal hydrologic changes in the development of the revised standard project flood for relatively small river basins are the assumption of wet antecedent conditions and the application of a higher unit hydrograph. Use of higher unit hydrographs, needed to evaluate the changed hydraulic conditions previously described, and the same standard project storm, resulted in revised standard project floods slightly larger than the August 1955 discharges.

For large river basins, or any basin where because of large natural storage the volume of runoff has considerable influence on the magnitude of the peak, it may be necessary to increase the amount of standard project rainfall for the longer durations. The amount of this increase will depend largely

on the judgment of the hydrologist and engineer with careful weighing of the basin characteristics, its susceptibility to the "Diane"-type storm, and the degree of protection justified for the area to be protected. In the Farmington River, a tributary of the lower Connecticut River, the design storm was assumed to be a combination of the previous standard project storm for short durations but extended to the depth-area-duration relationships for "Diane" for the longer durations. It was necessary to make this adjustment for the Farmington River because the extensive flood plains in the lower part of the watershed cause maximum stages to be a function of flood volume rather than peak inflow. Similar adjustments in the standard project storm should be made when required for specific problem areas.

Peak Discharge Frequencies

Peak discharge frequencies are used by the Corps of Engineers primarily to make an economic evaluation of proposed flood control measures. Since the prime purpose of the measures are protection against the major floods it is necessary to ascertain the frequency of the larger floods. This requires considerable extrapolation of frequency curves which have been derived from relatively short periods of record. Hence the procedure and accuracy of the extrapolation has considerable influence on the economic justification of the project.

For many years the New England Division of the Corps of Engineers used the frequency formula:

$$F = \frac{n}{m - 0.5}$$

where F = probable frequency in years
n = total numbers of years of record
m = number of times peak was equaled or exceeded during period of record.

The plotting positions of the annual and more frequent floods produced a fairly well defined curve, but the extended curve rarely went through the plotted points of the few major floods. This condition was particularly true for the stations with short records. It was then necessary to extrapolate the curve to the range of the rare floods. Considerable personal judgment was required in this extrapolation, and the accuracy of the final curve was problematical.

More recently the New England Division adopted the method devised by L. R. Beard* which assumes that logarithms of the annual peak discharges are normally distributed, and that statistical procedures are applicable. This method permits regional analyses and a correlation between short and long period records. Except for selection of stations for correlating purposes, and the applicable skew factor, the curves are determined analytically, thus minimizing the personal judgment in plotting and extrapolating data. It was considered that this procedure was most suitable for the needs of the Corps of Engineers to provide consistent frequency analyses for economic studies.

Following the record floods it was necessary to review and revise the

* Estimation of Flood Probabilities, Separate No. 438, May 1954.

frequency curves derived in 1950 using the 1955 flood peaks plus five additional years of record. The additional period of record had an appreciable effect on the frequency as shown on Plates 3 and 4, but further change was made because computations indicated an increase in the skew factor from 0.3 to 1.0. On former curves, the August 1955 flood had frequencies of up to 10,000 years, and in some cases were well beyond the limit of the curves. The revised curves, with the increase in the skew factor, dropped the frequency of the 1955 flood to 100 to 400 years, depending on the specific location. Previous changes in frequency relationships were made in 1936 and 1938 following record floods in those particular years, so in the past 20 years there have been at least five revisions in flood frequencies.

For some time the writer has been questioning the procedures for developing flood frequency relationships, for the apparent infrequency of large floods do not appear consistent with the records. Furthermore, the fact that every time a major flood occurs the frequency relationship has to be revised is a source of annoyance, and ample reason for doubting any frequency relationship.

Frequency difficulties are common with all methods, for they all provide reasonable and similar peak discharge for the annual, 5-year, and 10-year floods, but for the more infrequent floods they commence to diverge. For floods over 100 years in frequency the variation is dependent on the method of plotting and extrapolation. The writer has considered that flood frequencies for stations with 100 years of record will give fairly reliable frequencies up to 50 years. Beyond this point the reliability drops rapidly and beyond 100 years the frequency is fictitious.

Many of the methods for determining flood frequencies are based on analyses using the maximum annual flood, assuming that if the record is long enough, the array of data will fall into a normal distribution and be subject to the laws of pure statistics. It is found in many methods that compiling an array of the annual events leads to an overwhelming number of small floods that have considerable influence on the plotting position of the large floods.

The advantage in selecting the annual event is the convenience in determining the probability of occurrence in any one year, and its use in economic analysis where cost of projects are expressed in annual costs. However, it is possible to obtain different frequency relationships (depending on the method) if a different time base is adopted for the array instead of the annual—say monthly, 6 months, or 2 years, 5 years, or even a decade. Using a decade for example eliminates the many smaller floods and places more weight on the larger floods.

The writer is convinced that large floods and the smaller annual events are not related and should not be compiled in the same array and analyzed by statistical methods. In other words, the meteorological and hydrological conditions that produce the larger floods may be quite different from those producing the smaller floods. Statistics concerning dice and cards are "pure" mathematics for there are definite limits in the sides of the dice and the number of cards. Floods, however, involve an uncertain time scale, and unknown limitations, hence the difficulties of "pure" mathematics. It is believed that analysis of floods should be divided into two categories: one for moderate annual floods, and the second for the larger floods. Possibly the decade flood is a reasonable time basis for the large floods, but obviously application of this time basis can be made only to stations with extremely long records.

In summary the occurrence of the record-breaking floods in 1955 caused a

major revision in the flood-frequency relationships in New England, as used by the Corps of Engineers. The many revisions in flood frequencies during the last 20 years raises considerable doubt and skepticism concerning the reliability of flood frequency analyses. Further research and thought is required on this subject to obtain reasonable and consistent frequency relationships that may be used with confidence for economic analyses.

Storage Requirements in Flood Control Reservoirs

Prior to 1955 it was considered there should be sufficient storage capacity in a flood control reservoir to hold 6 inches of runoff from the watershed upstream of the project. This rule-of-thumb developed from studying floods of record and determining the storage required to restrict reservoir releases to non-damaging bankfull flows. In general, 6 inches of storage provided a reasonably high degree of protection, but some projects required more storage, depending on the location of the project and the runoff characteristics of the watershed.

The 1955 floods produced not only noteworthy peak flows, but produced record-breaking volumes of runoff. Plate 5 shows the volumes of runoff, expressed in inches of depth from the watershed, for the 1936, 1938, and 1955 floods at selected stations near the storm centers. The March 1936 floods produced a high volume of runoff, but this occurred from two storm periods: the first occurring on March 12, 1936, and the second on March 19, 1936. Hence, the volume of runoff caused by rain, augmented by melting snow, is measured over a period of 13 days. In most areas a reservoir having a capacity of 6 inches with proper regulation procedures could adequately control this flood as well as the floods occurring in 1927, 1938, 1948, and the standard project flood.

The 1955 floods produced 10 to 12 inches of runoff in many areas, with most of this runoff occurring in one or two days. In fact, analysis of the records at several river gages show that 6 to 7 inches of runoff were experienced in 24 hours, and most of the remainder in 48 hours. It is essential on the basis of this larger volume of runoff to reappraise the storage requirements for flood control reservoirs.

Regulatory procedures for flood control reservoirs in New England vary depending on the flood development of the river basin, locations of other projects, and the communities to be protected. Knowledge of flood timing is very important for many reservoirs are regulated to desynchronize the discharge contribution on the controlled stream from the uncontrolled streams. Thus experience has shown there is no simple rule for selecting the necessary amount of flood control storage to satisfy all projects.

In general, the volume of runoff experienced in the 1955 floods has demonstrated that it is desirable to provide more than 6 inches of storage. In many of the projects now under construction in New England, or in the planning stages, a storage capacity equivalent to 8 inches, or more, of runoff from the drainage area of the project is being considered. Although the 1955 storms did not affect Vermont and New Hampshire, past storms, notably the hurricane of September 1938, crossed these states, and thus it is considered that reservoirs in these areas should also have more storage than previously believed necessary.

Additional storage in the flood control reservoirs is also essential to

provide not only for the single large flood, but to have a reserve for a sequence of minor flood events. Experience during the past 15 years in regulating reservoirs has shown on numerous occasions that flood-producing storms have the faculty of repeating themselves within a few days. Hence, storage from the first runoff may be retained in the reservoir, thus reducing the effectiveness of the reservoir for the second flood runoff.

The majority of reservoirs in New England have been operated by complete closure of the outlet gates during the flood period whenever the safe downstream channel capacity has been exceeded. Ordinarily this procedure provides the optimum effectiveness of the project. Because of the 1955 floods, however, it is necessary to review this regulating procedure, especially for the reservoirs with 6-inch capacities. For these large floods, it may be found that the greatest downstream reductions can be obtained by only partially closing the gates during the flood. A regulated release from the reservoir might avoid the rapid filling of the reservoir resulting in a large uncontrolled discharge over the spillway. Such a change in the presently prescribed operating procedure requires a careful review of each project to evaluate the inherent regulating problems of that specific project and river basin.

Interior Drainage and Pumping Capacity

Communities along the Connecticut River in Massachusetts and Connecticut were the only areas in New England affected by the 1955 floods where extensive local protection projects have been completed. Prior to 1955, analysis of the flood history in the Connecticut River led to the conclusion that coincident rainfall and flood stages were a remote possibility. Former major floods developed either from widespread storms like the 1936 and 1938 occurrences, or from an upstream storm like 1927. Because of the length of the basin these floods crested in the lower basin a day or two after the flood-producing storm, and hence there was no good record of coincident rainfall and flood stages. It was considered unlikely that a major flood on the lower Connecticut River could be produced entirely by the downstream tributaries, and more unlikely to have the flood-producing storm occur concurrently with the flood. It was necessary to synthesize concurrent rainfall and river stage conditions for the design of interior drainage and for selecting the capacity of pumping stations.

Plates 6 and 7 show the actual experiences in 1955 at Hartford, Connecticut and Springfield, Massachusetts. It is obvious from an inspection of these plates that there must have been considerable interior runoff and concurrent flood stages, and records show many pumping stations were operated at full capacity during the flood. Insofar as known, the existing pumps were able to keep up with the inflow, although there were numerous locations where damage occurred within the protected area from inadequate municipal drainage facilities. It is possible that the flow in more adequately-sized drains would have overtaxed the pumping capacity.

Plate 8 shows a curve developed in 1945 for the design of interior drainage and for determining pump capacity for two projects in West Springfield. This curve is an envelope of many points denoting various probabilities of river stages and estimated concurrent rainfall intensities for each month of the year.

Plate 8 also shows plotted points for hourly rainfall values and concurrent river stages at Springfield, Massachusetts and a curve enveloping these

points. Compared with the former criteria the August 1955 storm produced more critical conditions than previously considered for design purposes. Future hydrologic criteria for determining interior drainage and pumping capacities will certainly include the conditions occurring in 1955. A new design curve of coincident rainfall and flood stages will be developed which will envelop the 1955 points and have greater rainfall rates at the higher stages than previously assumed. Selected pumping capacity, however, for any specific project will depend on economic justification and the pumping needs for the particular location.

Transposition of "Diane" Storm

The Corps of Engineers has been directed to review the flood control program in the northeastern states in order to evaluate its adequacy for the "Diane"-type flood and to determine whether additions or modifications are required in the program. In this review it is necessary to study not only the areas affected by the 1955 floods, but all other areas susceptible to this type of storm. This leads to the subject of storm transposition and whether it is proper to transpose the storm to other areas without modification.

The Hydrologic Services Division of the U. S. Weather Bureau has made comprehensive studies of all prior notable storms of record to determine their synoptic characteristics, and the effect of topographic features and location on the moisture availability. The derivation of the probable maximum precipitation, as previously described and used in developing the spillway design flood, was one of the results of this study. Generalized charts in reports prepared by the U. S. Weather Bureau show the probable maximum precipitation for various zonal subdivisions of the United States. Plate 9 shows a generalized chart for the New England-New York area, as used in Hydrometeorological Report No. 28.

Except for northern Maine, New Hampshire, and Vermont, the probable maximum precipitation for a 24-hour period exceeded the "Diane"-storm for a comparable duration, hence it is considered meteorologically possible for the storm to be transposed to most of the northeastern states. However, the studies by the Weather Bureau indicated that the isohyetals of the probable maximum precipitation progressively decrease in a northerly direction. Thus, moving the storm over the northern states without modification would be changing the probability of its occurrence. For example, superimposing the depth-area-duration relationship of the "Diane"-storm over central Vermont without some decrease would be producing a synthetic storm condition for that particular area of greater infrequency than the actual event.

It is considered proper to assume that a transposed, "Diane"-storm should have approximately the same probability of occurrence as the actual storm. To accomplish this is the New England area it is assumed that the storm can be transposed anywhere south of the Vermont-New Hampshire-Massachusetts line without modification. North of this state line the rainfall amounts and intensity are decreased in proportion to the 24-hour isohyetals shown on the generalized estimates of the probable maximum precipitation. Using this procedure the depth-area-duration amounts of the transposed "Diane"-storm to mid-Vermont and New Hampshire for the all-season envelope conditions would be decreased 12 per cent. The storm transposed over the northern parts of these states for this same condition would be decreased from 20 to 25 per cent.

The synthetic "Diane"-type flood, derived by storm transposition will supplement the standard project flood to test the flood potential of any area, and the flood control effectiveness of proposed projects. Whether or not projects will be recommended to protect against such a severe flood is a matter of economic justification.

Discussion

This paper has described the effect of the August 1955 storms and floods on those particular phases of hydrologic design criteria which have been involved in recent studies by the Corps of Engineers in New England. Very likely there are other features that have not yet been considered merely because investigations to date have not required their application. Previously the March 1936 and September 1938 storms and floods were used as "yardsticks" to test the hydrologic requirements for flood control projects. The phenomenal conditions of 1955 in many ways surpassed these two previous floods and today the 1955 rainfall and discharge amounts are being commonly quoted and used in flood control investigations.

So far the emphasis has been on studies of "Diane", principally because of the record-breaking rainfall and the widespread devastation caused by this storm. Although overshadowed by "Diane", the storm from 14-17 October 1955 ranks with the most severe in the northeastern states. Record-breaking floods occurred in southwestern Connecticut in areas previously spared by "Diane". Additional research and studies are necessary to fully appraise the magnitude of this meteorological event.

The occurrence of three major storms within two months in 1955 re-emphasizes the fact that great storms are not just freaks of nature and of such rare and infrequent occurrences that they "can't happen again." Similar storms can occur again—when, no one can predict—and the planners of the future must recognize this fact.

TABLE 1

MAXIMUM RECORDED RATES OF RAINFALL DURING STORMS OF 1955

IN INCHES

Storm & Location of Storm Center	Hours 1	2	3	6	12	24	48	Total Storm
August 11-14, 1955								
Slide Mt., N.Y.	1.60	3.00	4.15	6.62	9.85	11.42	12.57	15.15
August 17-20, 1955								
Mendon, Mass.	3.20	5.20	5.86					
W.Brookfield, Mass.				7.9				
Westfield, Mass.					11.7	18.2	19.8	19.8
October 14-17, 1955								
Brentwood, L.I.	2.15	3.20	4.10	6.15	7.53	8.88		
Stamford, Conn.							11.94	13.29

TABLE 2

MAXIMUM AVERAGE DEPTH OF RAINFALL IN INCHES

Area in Square Miles	DURATION OF RAINFALL IN HOURS 6	12	18	24	36	48	72	96
11-15 August 1955								
10	5.6	8.8	10.3	10.8	12.0	12.5	13.5	14.5
100	4.4	6.9	8.0	8.4	11.1	12.0	12.7	13.1
200	3.9	6.0	6.9	7.5	10.5	11.6	12.3	12.5
500	3.3	4.8	5.6	6.5	9.7	10.7	11.3	11.5
1,000	3.0	4.1	5.0	6.0	9.0	9.8	10.4	10.6
2,000	2.6	3.6	4.6	5.5	8.4	9.0	9.6	9.8
5,000	2.1	3.0	4.0	5.0	7.5	8.1	8.7	9.0
10,000	1.7	2.6	3.6	4.5	6.6	7.2	8.0	8.4
17-20 August 1955								
Max. Station	7.9	11.7	14.3	18.2	19.6	19.6	19.8	
10	7.8	11.1	13.0	16.4	18.9	19.4	19.6	
100	7.6	10.5	11.6	14.6	18.1	18.8	19.0	
200	7.4	10.2	11.4	14.2	17.6	18.2	18.4	
500	6.8	9.7	10.8	13.4	16.8	17.2	17.3	
1,000	6.2	9.2	10.2	12.4	15.9	16.2	16.4	
2,000	5.4	8.0	9.4	11.2	14.5	14.9	15.2	
5,000	4.0	6.3	7.9	9.5	12.1	12.6	13.0	
10,000	3.1	5.0	6.5	8.0	10.0	10.6	10.8	
Standard Project Storm (a) (Centered at Westfield, Mass.)								
10	9.1	10.3		11.4		12.8	13.5	13.7
100	8.2	9.3		10.3		11.7	12.3	12.6
200	7.8	8.7		9.7		11.2	11.7	12.0
500	7.0	7.9		8.8		10.3	10.8	11.1
1,000	6.4	7.2		8.0		9.5	10.1	10.4
2,000	5.8	6.5		7.2		8.6	9.2	9.5
5,000	4.7	5.3		5.9		7.4	7.9	8.9
10,000	3.9	4.4		4.9		6.3	6.9	7.3
Probable Maximum Precipitation (Centered at Westfield, Mass.)								
10	22.8	25.2		27.3		29.1		
100	17.0	19.9		22.2		23.4		
200	15.2	18.2		20.5		21.8		
500	12.7	15.8		18.0		19.3		
1,000	10.7	13.7		16.0		17.4		

(a) Generally applicable for basins less than 1,000 square miles.

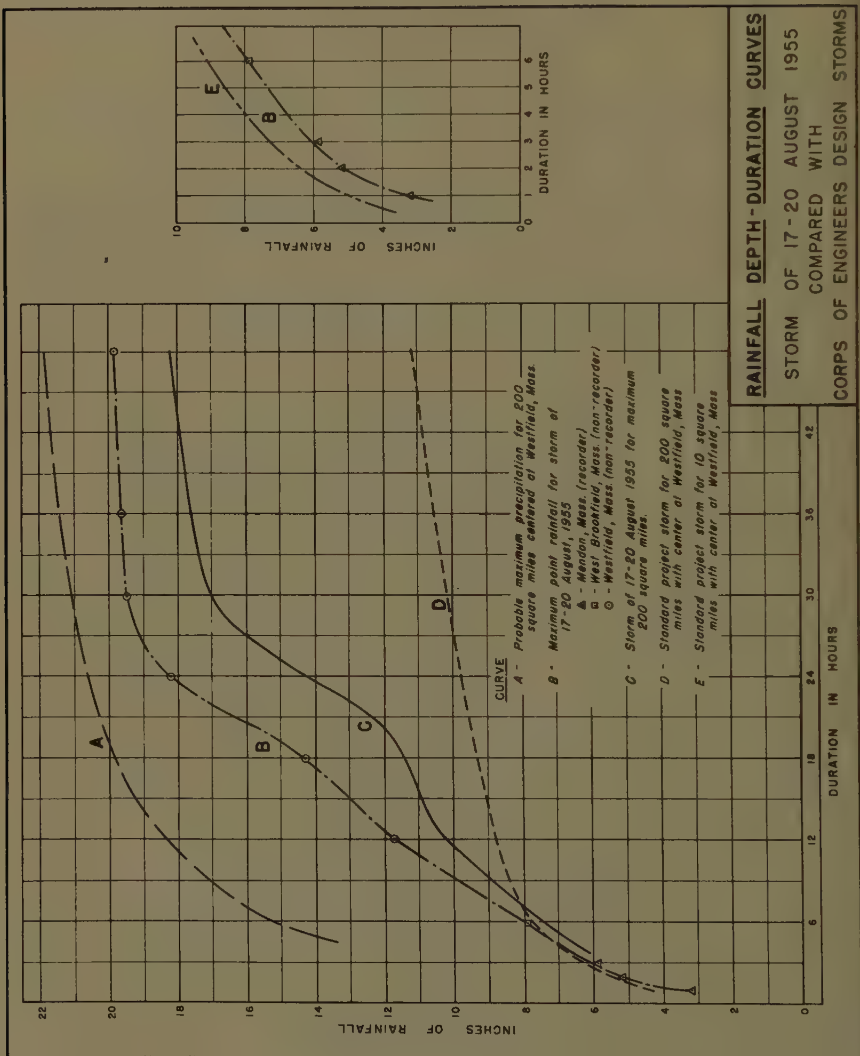
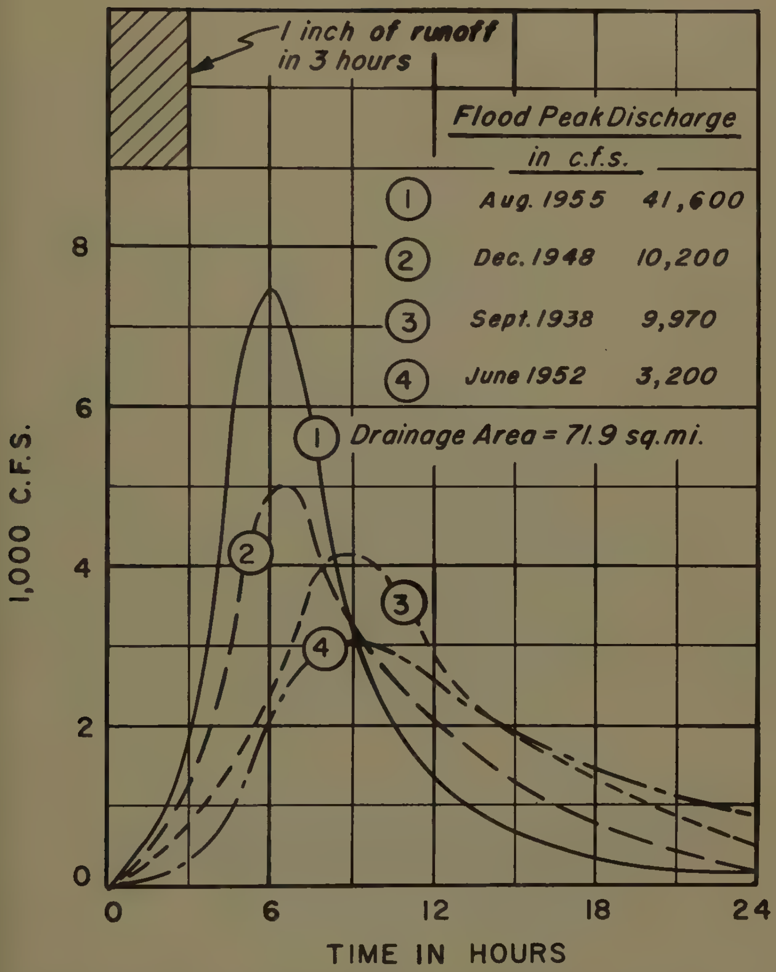
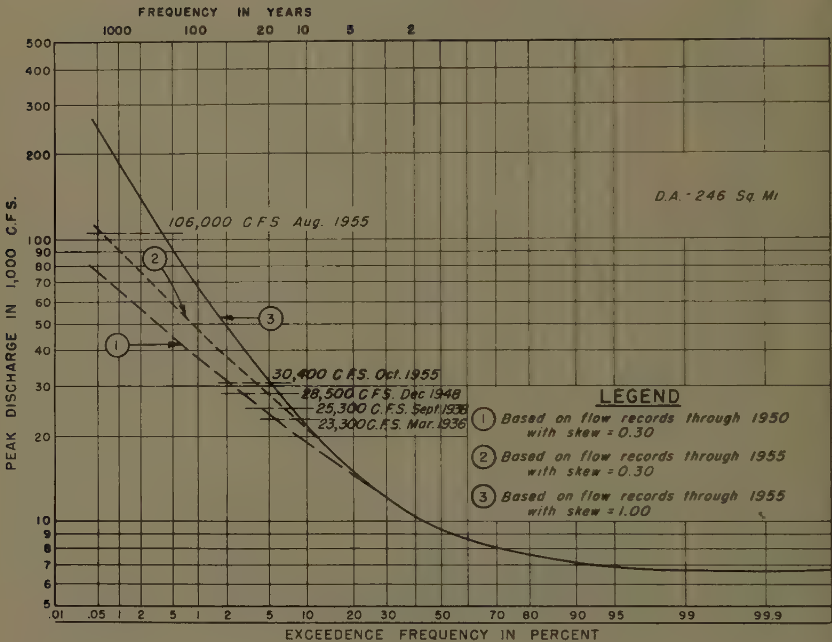

RAINFALL DEPTH-DURATION CURVES
STORM OF 17-20 AUGUST 1955
COMPARED WITH
CORPS OF ENGINEERS DESIGN STORMS
CURVE
A - Probable maximum precipitation for 200 square miles centered at Westfield, Mass.
B - Maximum point rainfall for storm of 17-20 August, 1955
▲ - Mendon, Mass. (recorder)
⊡ - West Brookfield, Mass. (non-recorder)
⊙ - Westfield, Mass. (non-recorder)
C - Storm of 17-20 August 1955 for maximum 200 square miles.
D - Standard project storm for 200 square miles with center at Westfield, Mass
E - Standard project storm for 10 square miles with center at Westfield, Mass
INCHES OF RAINFALL
DURATION IN HOURS
DURATION IN HOURS
INCHES OF RAINFALL
A
B
C
D
E
B

PLATE NO. 1



NAUGATUCK RIVER-THOMASTON CONN.
UNIT HYDROGRAPHS

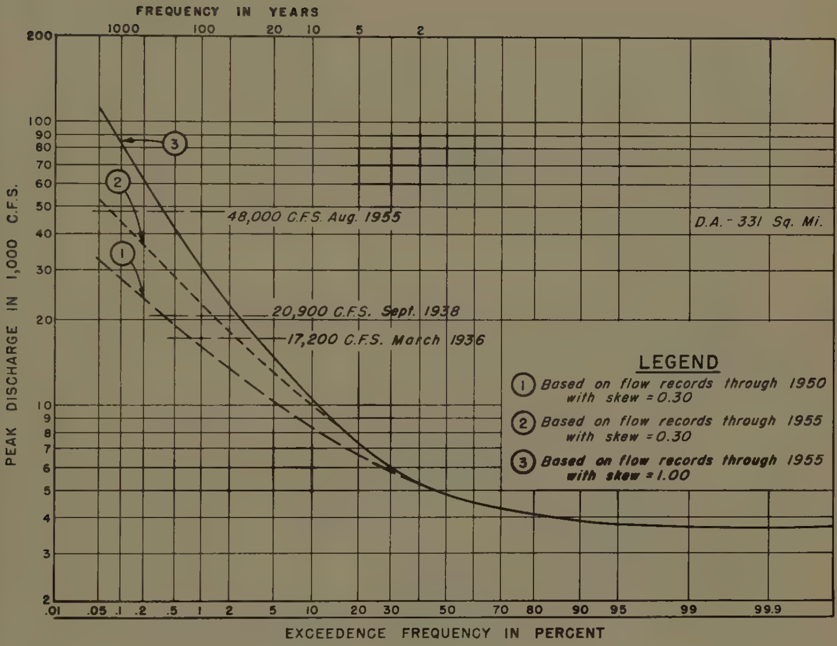
PLATE NO. 2



NAUGATUCK RIVER - NAUGATUCK, CONNECTICUT

EFFECT OF THE 1955 FLOOD ON DISCHARGE FREQUENCY CURVE

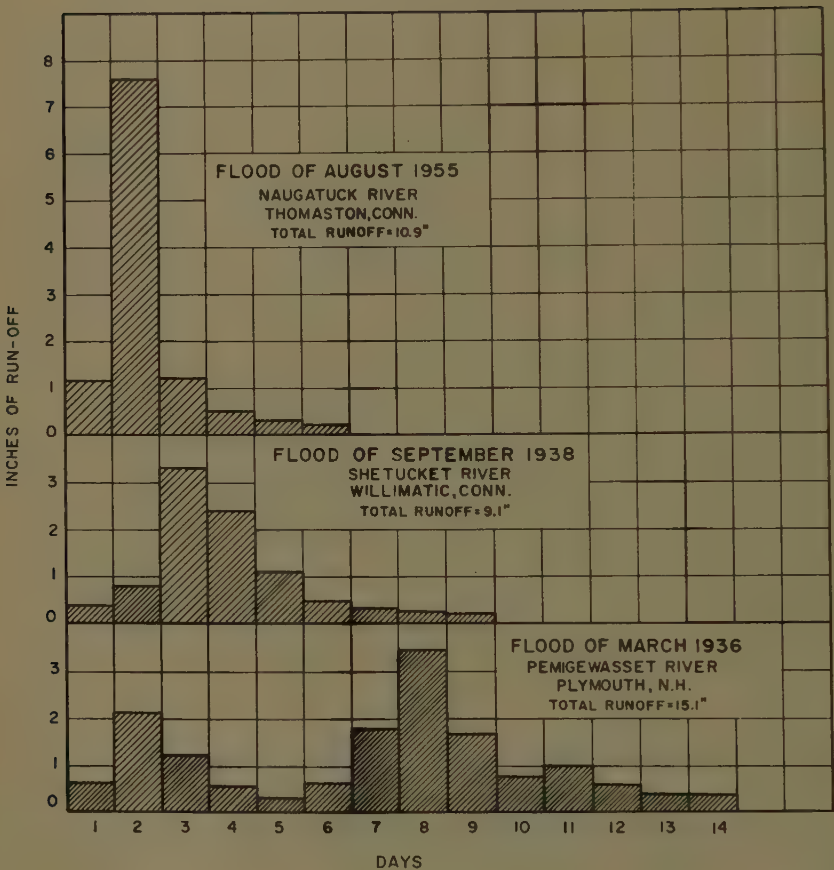
PLATE NO. 3



QUINEBAUG RIVER - PUTNAM, CONNECTICUT

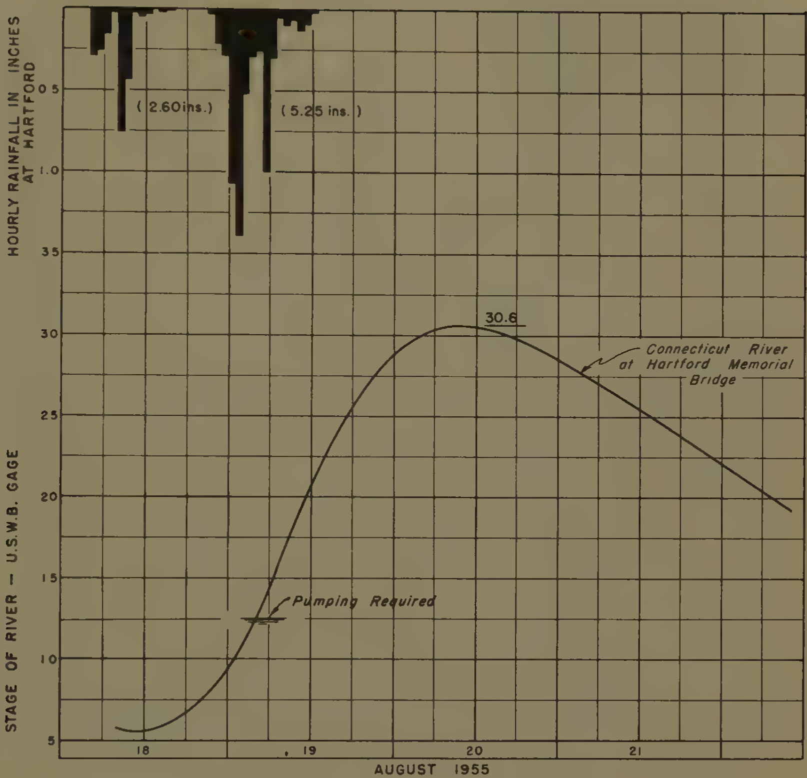
EFFECT OF THE AUGUST FLOOD
ON DISCHARGE FREQUENCY CURVE

PLATE NO. 4



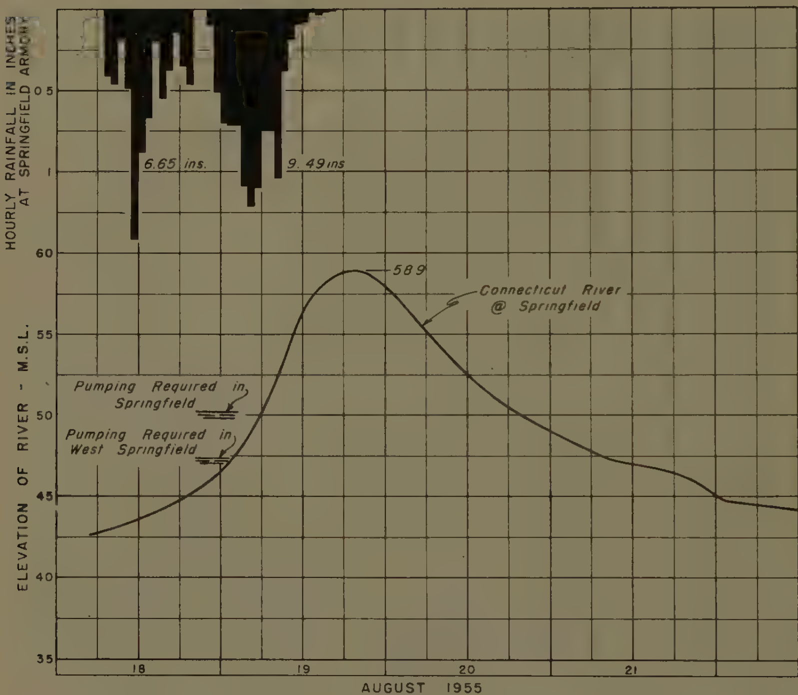
FLOODS OF RECORD IN NEW ENGLAND
VOLUME OF FLOOD RUNOFF AT SELECTED GAGES

PLATE NO. 5



CONNECTICUT RIVER – HARTFORD, CONNECTICUT
COINCIDENT RAINFALL AND FLOOD STAGES
AUGUST 1955

PLATE NO. 6

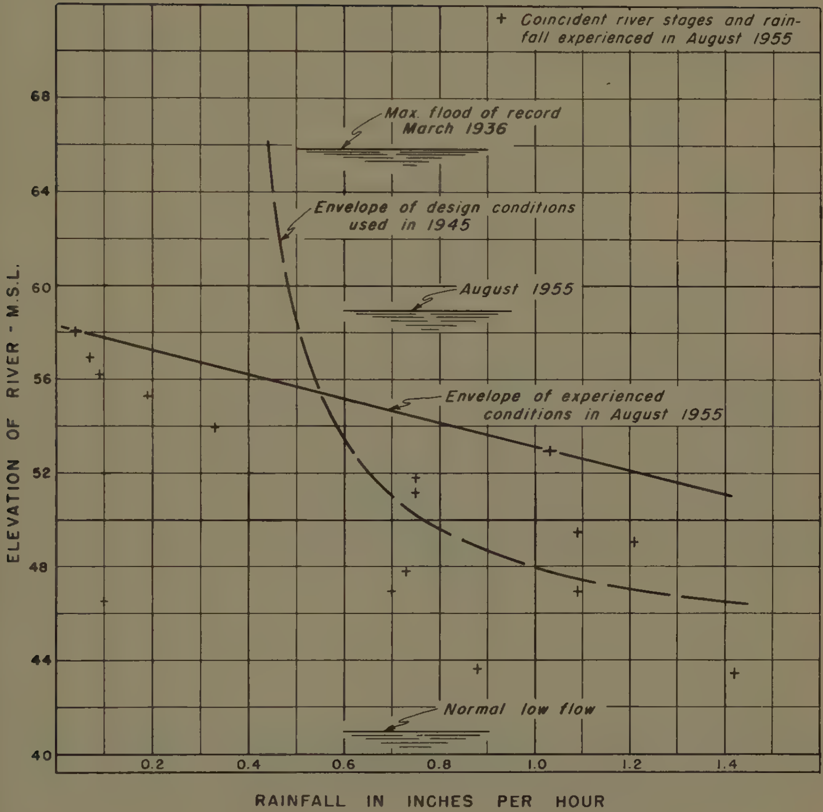


CONNECTICUT RIVER SPRINGFIELD, MASSACHUSETTS

COINCIDENT RAINFALL AND FLOOD STAGES

AUGUST 1955

PLATE NO. 7



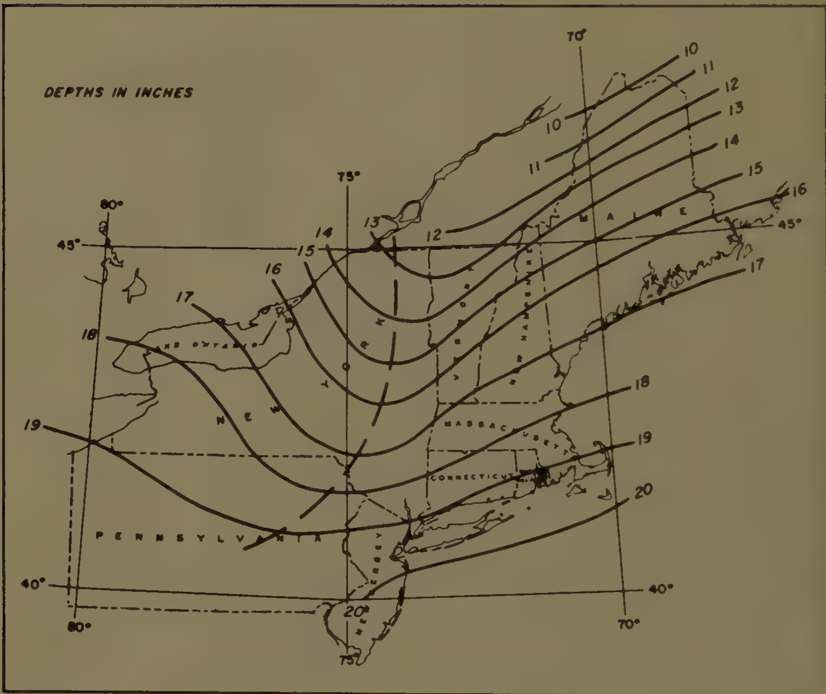
CONNECTICUT RIVER – SPRINGFIELD, MASSACHUSETTS

COINCIDENT FLOOD STAGES AND RAINFALL FOR DETERMINATION OF INTERIOR DRAINAGE AND PUMPING CAPACITY

PLATE NO. 8

U. S. WEATHER BUREAU

Hydrometeorological Report No. 28



GENERALIZED ESTIMATE
OF PROBABLE MAXIMUM PRECIPITATION
24 HOURS--500 SQUARE MILES

PLATE NO. 9

Journal of the
HYDRAULICS DIVISION
Proceedings of the American Society of Civil Engineers

SKIN FRICTION EXPERIMENTS ON ROUGH WALLS[a]

G. M. Sacks[1]
(Proc. Paper 1664)

SUMMARY

It is shown that effects of roughness on the skin friction of a flat plate at very high Reynolds numbers can be investigated by means of experiments in a pipe at moderate Reynolds numbers, provided the surface roughness of the flat plate can be reproduced exactly in the pipe. Some experiments on these lines are described, using air at speeds up to 270 ft. per sec. in a pipe of 2 in. diameter. It was found that the effect of roughness consisting of widely spaced peripheral wires in the pipe could be correlated with the drag of a circular cylinder on a plane wall, as obtained from the measured pressure distribution around the cylinder.

INTRODUCTION

The effect of roughness on the resistance of a body moving through a fluid is of importance in several branches of engineering. For example, roughness has a considerable effect on the resistance of a ship, even for the smoothest forms of hull that are now used, and there may also be important effects on aircraft and in hydrodynamic machines. In some of these cases, notably ships and aircraft, the Reynolds numbers are very large.

The problem is a complex one because of the large number of parameters required to specify the nature of a rough surface. Moreover, no satisfactory method has yet been developed for measuring directly the local shear stress on a rough wall and it is only in the case of flow through a pipe of constant

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a. Abbreviated version of thesis presented to the Univ. of Cambridge, England, in partial fulfilment of the requirements for the degree of Master of Science.

1. National Mech. Eng. Research Inst., Council for Scientific and Industrial Research, South Africa.

section that the effects of roughness can be measured simply.

The effect of roughness on the overall resistance of a flat plate can be investigated by measurements in a wind tunnel, water channel, or towing tank, but there are difficulties in deriving local values of skin friction coefficient from the overall resistance. Moreover, with all these methods the maximum Reynolds number is limited by the size of the apparatus and the speed that can be obtained.

Much of the previous work on the effects of roughness has been based on the classic experiments of Nikuradse,(1) in which closely packed sand grains were used to form the rough surface. Nikuradse investigated the resistance of pipes roughened in this way, over a wide range of relative grain size and Reynolds number. Subsequent work has usually been concerned with establishing an "equivalent sand roughness" for each surface considered, to enable the resistance to be obtained from Nikuradse's results. Of particular interest are the experiments of Schlichting,(2) in which the equivalent sand roughness was determined by measurements in a rectangular channel, for a variety of regular roughness patterns.

It is now clear, however, that for many forms of roughness of practical interest it is not possible to define satisfactorily an equivalent sand roughness. In particular, in most practical cases, the roughness elements are of non-uniform size, so that the "completely rough" regime obtained by Nikuradse in which the skin friction is independent of Reynolds number and depends only on the relative roughness, is not found. For example Todd(3) has shown that ships' hulls are hardly ever "completely rough" in this sense.

The procedure to be described in this paper was suggested by Professor J. H. Preston. The basis of the method is that in turbulent flow in a pipe or boundary layer there is a region near the wall in which $\frac{u}{u^*}$ is a universal function of $\frac{u^*y}{\nu}$, where u^* is the "friction velocity," u is the mean velocity at a distance y from the wall, and ν is the kinematic viscosity. Since the functional relation between $\frac{u}{u^*}$ and $\frac{u^*y}{\nu}$ is the same for a pipe as for a boundary layer, the conditions near the wall in a pipe can be related to conditions in a boundary layer. This enables the increase of skin friction due to roughness on a flat plate at a high Reynolds number to be determined from experiments with air in a pipe of only a few inches diameter. The roughness to be investigated must be reproduced exactly on the internal surface of the pipe, and to allow this to be done the pipe is constructed so that it can be opened by splitting it along its length. With roughness of representative height k, the air speed in the pipe is adjusted to give the same value of the similarity parameter $\frac{u^*k}{\nu}$ as on the full-scale flat plate. If the roughness height k is the same in the two cases, the requirement for similarity is simply that $\frac{u^*}{\nu}$ should be the same.

When these experiments were started, the intention was to investigate the effects on skin friction of various forms of paint roughness. Great difficulty was experienced, however, in reproducing satisfactorily in the pipe the required form of rough surface, since this depended not only on the type of paint but also on the method of application and the nature of the surface before painting. Attention was therefore concentrated on a simple form of roughness, which could be accurately specified and reproduced and would

consist of widely spaced excrescences as in most forms of paint roughness. This was formed by wires of small diameter, uniformly spaced and lying on the surface perpendicular to the flow. Measurements were made for a constant wire diameter at various values of the wire spacing.

In order to clarify the results obtained with these wires, some measurements were also made of the pressure distribution around a circular cylinder lying on a flat plate, completely submerged in the turbulent boundary layer. The drag coefficient derived from these measurements could be correlated with the increase in friction coefficient caused by the wires in the pipe, making a small allowance for the change of shear stress at the wall in the immediate vicinity of the wire.

Notation

b	distance between consecutive wires
$C_D = \dfrac{\bar{D}}{1/2\rho u_d^2 d}$	drag coefficient for cylinder
$C_p = \dfrac{p - p_D}{1/2\rho u_d^2}$	pressure coefficient on cylinder
D	diameter of pipe
$\bar{D}$	drag per unit length of cylinder
d	effective diameter of wire or cylinder on wall
G	constant in equation (4)
$H = 10.3\ (G/D)^{14/13}$	
k	height of representative roughness element
n	exponent in equation (4)
p	local static pressure at point on cylinder
p_o	static pressure in boundary layer, undisturbed by cylinder
d_p	pressure drop in pipe due to one wire
d_{p1}, d_{p2}	components of d_p corresponding to β_1 and β_2
$R_p = \dfrac{u_m D}{\nu\rho}$	Reynolds number of pipe flow
$R_x = \dfrac{Ux}{\nu_f}$	Reynolds number of flat plate flow
U	velocity outside boundary layer, on flat plate
u	local velocity in pipe or boundary layer
$u^* = (\tau_o/\rho)^{1/2}$	friction velocity, for smooth surface
u_d	velocity in pipe or boundary layer at $y = d$, with no wire or cylinder on wall.

u_m	mean velocity in pipe
x	distance from leading edge of flat plate
y	distance from wall
β_1	increase of skin friction coefficient caused by drag of wires
β_2	increase of skin friction coefficient caused by change of shear stress at wall due to presence of wire
$\gamma_f = \dfrac{\tau_0}{1/2\rho U^2}$	skin friction coefficient on smooth flat plate
$\gamma_p = \dfrac{\tau_0}{1/2\rho u_m^2}$	skin friction coefficient in smooth pipe
$\Delta\gamma$	increase of skin friction coefficient due to roughness
ν	kinematic viscosity of fluid
ρ	density of fluid
τ_o	shear stress at smooth wall
Suffixes f	conditions on a flat plate
p	conditions in a pipe

Theoretical Basis of Experiments

As already mentioned, the method to be described enables the effect of roughness on the skin friction of a flat plate, at a high Reynolds number, to be determined by means of experiments in a pipe at a much lower Reynolds number. The method depends on the known existence of a region near the wall, in turbulent flow, in which u/u* in a universal function of $\frac{u^*y}{\nu}$. Provided all the roughness elements lie well within this region, for the same Reynolds number, the increase of skin friction due to roughness of given shape, and representative height k, may be assumed to be a function of k, u*, ρ and μ. The assumption has been confirmed directly by Hama,[4] from experiments in a pipe and a flume roughened artificially in exactly the same way, the value of $\frac{u^*k}{\nu}$ being the same in the two cases.

In comparing the effect of roughness in a pipe with that on a flat plate, the appropriate dimensionless coefficient to represent the increase of skin friction is $\Delta\gamma/\gamma$, where γ is the local skin friction coefficient at the smooth wall and $\Delta\gamma$ is the increase due to roughness. It is necessary to express the effect of roughness in this way, and not simply as $\Delta\gamma$, because the velocity used in defining conventionally the skin friction coefficient is not the same for a pipe as for a flat plate.

With the assumption discussed above, if k is small compared with the radius of the pipe, $\Delta\gamma/\gamma$ must be a function of $\frac{u^*k}{\nu}$, for a rough surface of given form, and this function will be the same for a pipe as for a flat plate.

Thus the effect of roughness on a flat plate can be investigated by making experiments in a pipe with the required value of $\frac{u^*k}{\nu}$. It will usually be convenient to make the roughnesses in the two cases identical, not only in form but also in scale. Using suffixes p and f to denote conditions in the pipe and on the flat plate, the conditions for dynamical similarity is then

$$\frac{u^*_p}{u^*_f} = \frac{\nu_p}{\nu_f} \tag{1}$$

u^*_p and u^*_f are defined here in terms of the smooth wall, but if Eq. (1) is then satisfied, the corresponding equation for the rough wall will also be satisfied. Thus to determine the conditions on a flat plate corresponding to given conditions in a pipe, or vice versa, it is only necessary to use Eq. (1), in conjunction with empirical laws giving the skin friction for smooth surfaces in the two cases.

For flat plates the Reynolds numbers that are of interest in this connection are large, and an appropriate law to use for the skin friction of the smooth surface is that given by Falkner:[5]

$$\gamma_f = 0.0262\ R_x^{-1/7},$$

where

$$\frac{\gamma_f}{2} = \frac{\tau}{\rho U^2} = \left(\frac{u_f^*}{U}\right)^2,$$

so that

$$u_f^* = 0.1145\ U\ R_x^{-1/14}. \tag{2}$$

For smooth pipes the usual Blasius law for skin friction is

$$\gamma_p = 0.0791\ R_p^{-\frac{1}{4}} \tag{3}$$

but, as pointed out by Schlichting[6] and by others, this gives good agreement with experiments only up to $R_p = 10^5$, and requires modification for higher Reynolds numbers. It is convenient in general to use a law of the Blasius type, expressed in the form

$$\gamma_p = 2\ G^2\ R_p^{-n}, \tag{4}$$

where G and n are constants, chosen to fit the available experimental results over the required range of Reynolds numbers.

Equation (4) gives

$$\frac{\gamma_p}{2} = \left(\frac{u_p^*}{u_m}\right)^2 = G^2\ R_p^{-n}$$

and hence

$$u_p^* = G\ u_m\ R_p^{-n/2} \tag{5}$$

Equations (5), (2) and (1) then give

$$\frac{u_p^*}{u_f^*} = \frac{G}{0.1145} \cdot \frac{u_m}{U} \cdot \frac{R_x^{1/14}}{R_p^{n/2}} = \frac{\nu_p}{\nu_f},$$

or $$\frac{u_m}{\nu_p} \cdot R_x^{1/14} = \frac{0.1145}{G} \cdot \frac{U}{\nu_f} \cdot R_p^{n/2} .$$

But $$\frac{u_m}{\nu_p} = \frac{R_p}{D} \text{ and } \frac{U}{\nu_f} = \frac{R_x}{x},$$

so that $$\frac{R_p \cdot R_x^{1/14}}{D} = \frac{0.1145}{G} \cdot \frac{R_x \cdot R_p^{n/2}}{x}$$

and hence $$R_x^{13/14} = 8.73\, G\, \frac{x}{D} \cdot R_p^{(1-n/2)} \tag{6}$$

It is more instructive to derive from Eq. (6) an expression which varies only slowly with R_p and x, for given D.

Thus $$R_x = H\left[x.R_p^{(1-n/2)}\right]^{14/13},$$

where $$H = 10.3\ (G/D)^{14/13}.$$

Hence $$\frac{R_x}{xR_p} = \frac{U}{\nu_f R_p} = H.x^{1/13}\ R_p^{(1-7n)/13} \tag{7}$$

Since H will be a constant for experiments in a given pipe, and since the right-hand side of Eq. (7) involves only small powers of x and R_p, the quantity R_x/xR_p will vary comparatively little over a wide range of x and R_p.

The values to be taken for the constants G and n in Eq. (4) must now be considered. In the experiments to be described in this paper the pipe Reynolds number R_p varied between about 10^4 and 3×10^5. The points in Fig. 1 show values of γ_p for smooth pipes, obtained by different experimenters over this range, as collected by Schlichting.(6) The broken line represents the Blasius law (Eq. 3) and agrees well with the experimental results only up to $R_p = 10^5$. Better agreement is obtained over the required range of R_p by using the equation

$$\gamma_p = 0.058\ R_p^{-0.22}, \tag{8}$$

i.e. by making $n = 0.22$ and $2G^2 = 0.058$ in equation (4), so that $G = 0.170$. Equation (8) is represented by the full line in Fig. 1.

For any given set of experiments with a pipe, the values of n and G should be chosen to fit the published experimental results as closely as possible over the required range of R_p. The method of using equation (7) to relate the pipe experiments to conditions on a flat plate will be illustrated by taking the pipe diameter and Reynolds number range of the present series of experiments, using the values of n and G corresponding to equation (8). The pipe diameter was 2 in., so that the values to be substituted in equation (7) are $D = \frac{1}{6}$ ft., $n = 0.22$, $G = 0.170$. Equation (7) then becomes

$$\frac{R_x}{xR_p} = \frac{U}{\nu_f R_p} = 10.5\ x^{1/13}\ R_p^{-0.0415} \qquad (9)$$

Fig. 2 shows $\frac{U}{\nu_f R_p}$ for the 2 in. diameter pipe, plotted against x for several values of R_p. This diagram shows that, for a wide range of conditions, the Reynolds number per foot on the flat plate is of order 10 R_p. Thus for large values of x the ratio $\frac{R_x}{R_p}$ is large, i.e. experiments in the pipe with a given roughness at fairly low Reynolds numbers enable the effects of the same roughness on a flat plate at very large Reynolds numbers to be predicted. It may be noted that the Reynolds number on the flat plate is independent of the kinematic viscosity ν_f, for given values of x and R_p, although the velocity U does of course depend on ν_f.

The maximum value of u_m in these pipe experiments was 270 ft. per sec., giving $R_p = 2.86 \times 10^5$ for air at standard atmospheric conditions. Fig. 2 then shows for example, that for x = 400 ft. $\frac{R_x}{x} = 2.84 \times 10^6$ ft.$^{-1}$ and $R_x = 1.13 \times 10^9$. For water at 15°C, $\nu = 1.23 \times 10^{-5}$ ft.^{2}/sec., so that these figures correspond to conditions on a ship's hull, 400 ft. from the bow, at a speed of 20.8 knots. Again, for the same conditions in the pipe, if x = 20 ft $\frac{R_x}{x} = 2.25 \times 10^6$ ft.$^{-1}$, so that $R_x = 45 \times 10^6$. Considering an aeroplane flying in the standard atmosphere, these figures correspond to conditions on the fuselage or wing, 20 ft. from the nose or leading edge, either at sea level at a speed of 209 knots or at 30,000 ft. at a speed of 463 knots. Lower values of R_x, corresponding to lower speeds of the ship or aeroplane, can of course be represented by using a lower air speed in the pipe, i.e. at lower value of R_p. The most convenient method of obtaining higher values would be to use a pipe of larger diameter.

To determine the total effect of a given roughness at a given speed on the drag of a ship or an aeroplane wing, it is necessary to find the values of $\Delta\gamma$ for the whole range of x from bow to stern or leading edge to trailing edge, and integrate to find the increase of drag. This means that experiments in the pipe are needed at several different values of R_p in order to keep the required constant value of $\frac{U}{\nu_f}$ over the appropriate range of x. Fig. 2 shows that the range of R_p required is not large, so that only 2 or 3 values of R_p will be needed in most cases.

A further question to be considered is the range of roughness height for which the method can be used with confidence. It is to be expected that the principal assumption on which the method is based will be valid provided all the roughness elements lie well within the region where $\frac{u}{u^*}$ is a universal function of $\frac{u^*y}{\nu}$. Preston[7] has shown that for a pipe this region extends to a distance from the wall equal to about 1/5 of the radius, while for a boundary layer the corresponding distance is about 1/5 of the boundary layer thickness, provided the pressure gradient is favourable or only slightly adverse. A

further condition to be satisfied, since the effect of curvature of the pipe wall has been neglected in the analysis, is that the roughness height should be small compared with the pipe radius. The latter condition is unlikely to impose any serious limitation in practice, since for the reason given earlier the roughness height should not be allowed to exceed about 1/10 of the pipe radius.

Unless the roughness is very small indeed, it will be necessary to apply a correction for the change of effective pipe diameter caused by the addition of the rough surface. In the experiments described here the effective pipe diameter was taken to be the diameter of a smooth pipe having a cross-sectional area equal to the mean value for the rough pipe.

Pipe Experiments

Apparatus

The pipe used for the experiments had an internal diameter of 2 in. and a total length of 24 ft. It consisted of three identical sections, each eight feet long, of which the first two served as the entry length necessary to obtain fully developed turbulent flow in the third section, the test length. Thus the distance from the pipe entry to the start of the test length was about 100 diameters.

Each of the 8 ft. sections was made from a pair of aluminum alloy bars, initially of rectangular section. A semi-circular channel of radius 1 in was machined in each of these bars, so that when the bars were bolted together they formed a smooth pipe of circular internal section. By unbolting the bars the pipe could be split open so that the internal surface could be roughened as required. Rubber sealing cord was used at all the joints, to prevent leaks when the pipe was closed. The pipe was accurately constructed so that there were no discontinuities at any of the joints.

In the pipe as originally constructed the whole of the interior surface was smooth, and the interior of the test length was hand finished to obtain a polished surface. The pipe was used in this state to obtain the basic friction for the "smooth" surface. This gave results in very close agreement with Eq. (8).

The pipe entry consisted of a wooden axi-symmetric fairing, followed by a toothed spoiler to ensure transition to turbulent flow at a fixed position. The downstream end of the pipe was connected, through an additional length of 2 in. diameter pipe and a diffuser, to the inlet of a three-state centrifugal blower, driven by a 10 H.P. motor. A simple adjustable shutter was used on the blower outlet to vary the flow.

The flow through the pipe was measured by a three-quarter radius flow meter as described by Preston,[8] placed in the additional length of 2 in. diameter pipe, about 2 ft. after the downstream end of the test length. The flow meter was calibrated by comparison with a traversed pitot tube, with an allowance for the effect of compressibility on the pitot-tube readings.

The gradient of static pressure along the test length was measured by means of static tubes, since static holes in the wall could not be used satisfactorily when the surface was roughened. The static tubes were calibrated by comparison with static holes in the smooth pipe.

The range of mean air speeds used in the pipe was from about 10 to 270 ft. per sec., giving values of R_p from about 10^4 to 2.9×10^5.

In deriving values of the friction coefficient from the measured pressure gradients along the pipe, a correction was applied for the effect of compressibility in causing an acceleration of the air in a pipe of constant cross-section. This correction altered the friction coefficient by as much as 9% at the highest speed used. The correction was calculated by means of one-dimensional theory, based on the mean velocity in the pipe.

Results

The original intention was to measure the increase of skin friction caused by the application of various paints to the interior surface of the pipe, and to use the results to estimate the effects of the paints on ship resistance. It was soon realised, however, that the additional roughness caused by the application of paint to a surface depends quite as much on the form of the surface before painting, and on the method of application of the paint, as on the nature of the paint used. Since the surface of the pipe was initially very smooth it was not found possible to produce a painted surface that had any direct practical interest. Moreover, no satisfactory method was found for measuring the roughness of a painted surface in exact geometrical terms.

For these reasons, the results obtained from the painted surfaces were not of much interest. The paints used had only very small effects on the skin friction, in a typical case $\frac{\Delta \gamma}{\gamma}$ was zero for $R_p < 0.7 \times 10^5$ and increased gradually at higher Reynolds numbers up to about 1/2% at $R_p = 2.6 \times 10^5$. Within the range of Reynolds numbers considered, there was no indication of any tendency to approach the "completely rough" regime found by Nikuradse, in which the friction coefficient is independent of Reynolds numbers.

Because of the difficulty of obtaining useful results from painted surfaces, it was decided to make some experiments on surfaces with easily controllable roughness properties. A simple form of rough surface was obtained by fixing wires of diameter 0.0092 in. circumferentially around the inside of the pipe, with uniform axial spacing. After sticking the wires to the pipe with Durofix, it was found that the total height of each wire above the pipe surface (i.e. the effective wire diameter d) was 0.010 in. The pressure gradient along the pipe was measured for the full range of speeds, for distances b between successive wires of 1, 2, 4, 6 and 8 in., giving values of the ratio b/d from 100 to 800. Fig. 3 shows the skin friction coefficients, plotted logarithmically against R_p. From this diagram, values of the friction coefficient increment $\Delta \gamma$ can be obtained, for a range of values of R_p and b/d. Extrapolation of the curves to the left shows that $\Delta \gamma$ is zero for values of R_p less than about 1.8 x 10^4. This corresponds to $\frac{u^* d}{\nu} \approx 5$ and is in agreement with the result obtained by Nikuradse for sand roughness. A further analysis of these results will be given later.

Experiments on Circular Cylinders in a Turbulent Boundary Layer

To assist in the analysis of the measurements in a pipe with circumferential wires fixed to the inner surface, an investigation was made of the pressure distribution around a circular cylinder, lying on a flat plate completely

submerged in the turbulent boundary layer. The measurements were made on the floor of an open-return wind tunnel at air speeds (outside the boundary layer) up to about 100 ft. per sec. The range of Reynolds number $\frac{(u_d d)}{\nu}$ was from about 1800 to 8000. The thickness of the boundary layer was roughly 2 in., and to ensure that the cylinders were well within the region where the velocity profile followed the universal "inner law," cylinder diameters of 1/4, 3/16 and 1/8 in. were used.

To ensure two-dimensional flow, each of the cylinders was fitted with a pair of end plates; one of these end plates incorporated a protractor, so that by rotating the cylinder the complete pressure distribution could be found, using a single static hole. The normal floor of the wind tunnel was replaced in this region by a smooth flat plate, and the cylinder was kept in close contact with this throughout the measurements.

As expected, the pressure distribution differed considerably from that found for a circular cylinder in an unbounded uniform stream. Over the range of Reynolds number considered there was little variation in the distribution of pressure coefficient; Fig. 4 shows the distribution at $\frac{u_d d}{\nu} = 6000$, compared with that observed at the same value of $\frac{Ud}{\nu}$ in unconfined flow. (The pressure coefficient C_p is defined in terms of u_d, the velocity at y = d in the absence of the cylinder).

A few measurements of total and static pressure near the floor of the wind tunnel were also made. These showed regions of adverse pressure gradient and reversed flow, both in front of and behind the cylinder.

As already explained, the dimensionless parameter used for correlating measurements of the effect of roughness on skin friction is $\frac{u^*k}{\nu}$. In the case of the pipe with wires attached to the surface, k is equal to the effective wire diameter d, and it is therefore desirable to express the results of the measurements on cylinders in a wind tunnel in terms of $\frac{u^*d}{\nu}$. To achieve this, the skin friction on the surface was measured, with no cylinder present, using the pitot-tube method suggested by Preston.[7] Hence, for each value of d, values of $\frac{u^*d}{\nu}$ could be plotted against the reference pressure difference normally used to obtain the tunnel speed.

A drag coefficient for the cylinder was defined as

$$C_D = \frac{\bar{D}}{\frac{1}{2}\rho u_d^2 d}, \tag{10}$$

where $\bar{D}$ is the drag per unit length of cylinder. Values of this coefficient, obtained by integration of the pressure distribution, are plotted against $\frac{u^*d}{\nu}$ in Fig. 5. The scatter is rather large, but there is no significant indication of any large variation of C_D with Reynolds number.

Analysis of the Results Obtained with Circumferential Wires in the Pipe

Regarding the wires in the pipe as a form of roughness it is clear that the shape of the roughness, but not its scale, is completely specified by the ratio b/d. Hence, in accordance with the discussion given earlier, $\frac{\Delta\gamma}{\gamma}$ will be a function of b/d and $\frac{u^*d}{\nu}$. The increase of friction coefficient due to the roughness may be expressed as

$$\Delta\gamma = \beta_1 + \beta_2 \tag{11}$$

where β_1 is the increase caused directly by the drag of the wires and β_2 is caused by the change of shear stress at the wall of the pipe. (It will be shown later that β_2 is negative).

Similarly, the pressure drop in the pipe due to each wire may be expressed as

$$dp = dp_1 + dp_2$$

where dp_1 and dp_2 correspond to the terms β_1 and β_2 in equation (11). The component dp_1, and hence β_1, can be estimated if the drag coefficient C_D, as defined in equation (10), is known. The drag per unit length of each wire is

$$\bar{D}. = \tfrac{1}{2}\rho u_d^2 d.\, C_D,$$

so that the pressure drop in the pipe due to the drag of one wire is given by

$$\frac{dp_1}{\frac{1}{2}\rho u_m^2} = 4\,\frac{d}{D}\cdot C_D\cdot\left(\frac{u_d}{u_m}\right)^2 \tag{12}$$

To proceed further it is necessary to determine $\frac{u_d}{u_m}$ from the known relation between $\frac{u}{u^*}$ and $\frac{u^*y}{\nu}$ for turbulent pipe flow.

In the laminar sub-layer, for which $\frac{u^*y}{\nu}$ is less than about 5,

$$\frac{u}{u^*} = \frac{u^*y}{\nu} \tag{13}$$

and hence

$$\frac{u_d}{u_m} = \frac{u^*}{u_m}\cdot\frac{u_d}{u^*} = \frac{u^*d}{\nu}\cdot\frac{u^*}{u_m} = \frac{u_m d}{\nu}\cdot\frac{\gamma}{2} = R_p\cdot\frac{d}{D}\cdot\frac{\gamma}{2} \tag{14}$$

For values of $\frac{u^*y}{\nu}$ greater than about 30, the universal log law can be used:

$$\frac{u}{u^*} = 5.5 + 5.75\log\frac{u^*y}{\nu}\,. \tag{15}$$

Thus,

$$\frac{u_d}{u_m} = \frac{u^*}{u_m}\cdot\frac{u_d}{u^*} = 5.5\left(\frac{\gamma}{2}\right)^{\frac{1}{2}}\left(1 + 1.05\log\frac{u^*d}{\nu}\right)$$

$$= 3.89\,(\gamma)^{\frac{1}{2}}\left[1 + 1.05\log\left\{\frac{d}{D}\cdot R_p\left(\frac{\gamma}{2}\right)^{\frac{1}{2}}\right\}\right] \tag{16}$$

For this case $\frac{d}{D} = \frac{1}{200}$ and the relation between γ and R_p is known from the experiments in the smooth pipe. Hence the relation between $\frac{u_d}{u_m}$ and R_p, as given by equations (14) and (16), has been plotted in Fig. 6. The approximate limits of validity of equations (14) and (16) are respectively $R_p = 1.8 \times 10^4$ and $R_p = 1.3 \times 10^5$, and an arbitrary smooth interpolation curve has been drawn for the intermediate range of R_p.

For the experiments with the wires in the pipe, $\frac{u^*d}{\nu}$ varied from about 3 to 60. For the experiments with cylinders on the floor of the wind tunnel the values of $\frac{u^*d}{\nu}$ were considerably greater, covering a range from about 70 to 400. Unfortunately it was not possible to cover the same range of $\frac{u^*d}{\nu}$ in the two cases, because of the difficulty of using very small cylinders on the floor of the wind tunnel. Nevertheless, it is of interest to proceed further with the analysis of the results, on the assumption that the drag coefficient of about 0.7, shown in Fig. 5, is also correct at lower values of $\frac{u^*d}{\nu}$. It will also be assumed that the wires in the pipe act independently of one another, so that the drag of each is the same as that of an isolated wire on a wall.

Using Fig. 6 and equation (12), values of $\frac{dp_1}{1/2\,\rho u_m^2}$ have been calculated for a range of values of R_p, and are plotted as the upper curve in Fig. 7. Hence, for each wire spacing, β_1 and β_2 were found, using the values of $\Delta\gamma$ given by Fig. 3. The values of $\frac{dp_2}{1/2\,\rho u_m^2}$ calculated from β_2, showed no significant variation with wire spacing, thus justifying the assumption that the wires acted independently of one another, within the range of spacings considered. The value of $\frac{dp_2}{1/2\rho u_m^2}$ was found to be negative and independent of R_p, as shown by the lower curve in Fig. 3. It is of course to be expected that dp_2 (and β_2) will be negative, since the shear stress at the wall must be negative in the region of reversed flow immediately before and after each wire. In view of the rather doubtful assumption that C_D has the constant value of 0.7, down to the small values of $\frac{u^*d}{\nu}$ occurring in the pipe, it is not possible to determine from these results whether β_2 is genuinely independent of R_p, or whether the apparently constant value shown in Fig. 7, is fortuitous.

As already explained, the results indicate that the wires acted independently of one another for all the values of $\frac{b}{d}$ considered. For any distribution of roughness elements satisfying this condition, it should be possible to calculate approximately the effect on skin friction, if the drag coefficients of the elements are known. Strictly, some allowance should be made for the β_2 term in equation (11), but Fig. 7 suggests that even if this is ignored the error in estimating the total roughness effect will not be very large.

CONCLUSIONS

1) The drag of a flat plate at high Reynolds numbers can be determined from experiments made in a pipe at much lower Reynolds numbers, provided the surface characteristics of the flat plate can be reproduced exactly in the pipe.

2) The drag coefficient of a circular cylinder resting on a plane wall, completely submerged in the turbulent boundary layer, is about 0.72 for values of $\frac{u^*d}{\nu}$ between 100 and 400.

3) The effects of widely spaced roughness elements on skin friction can be estimated satisfactorily if the appropriate drag coefficients are known, making a small allowance for the change of shear stress at the wall in the immediate vicinity of a roughness element.

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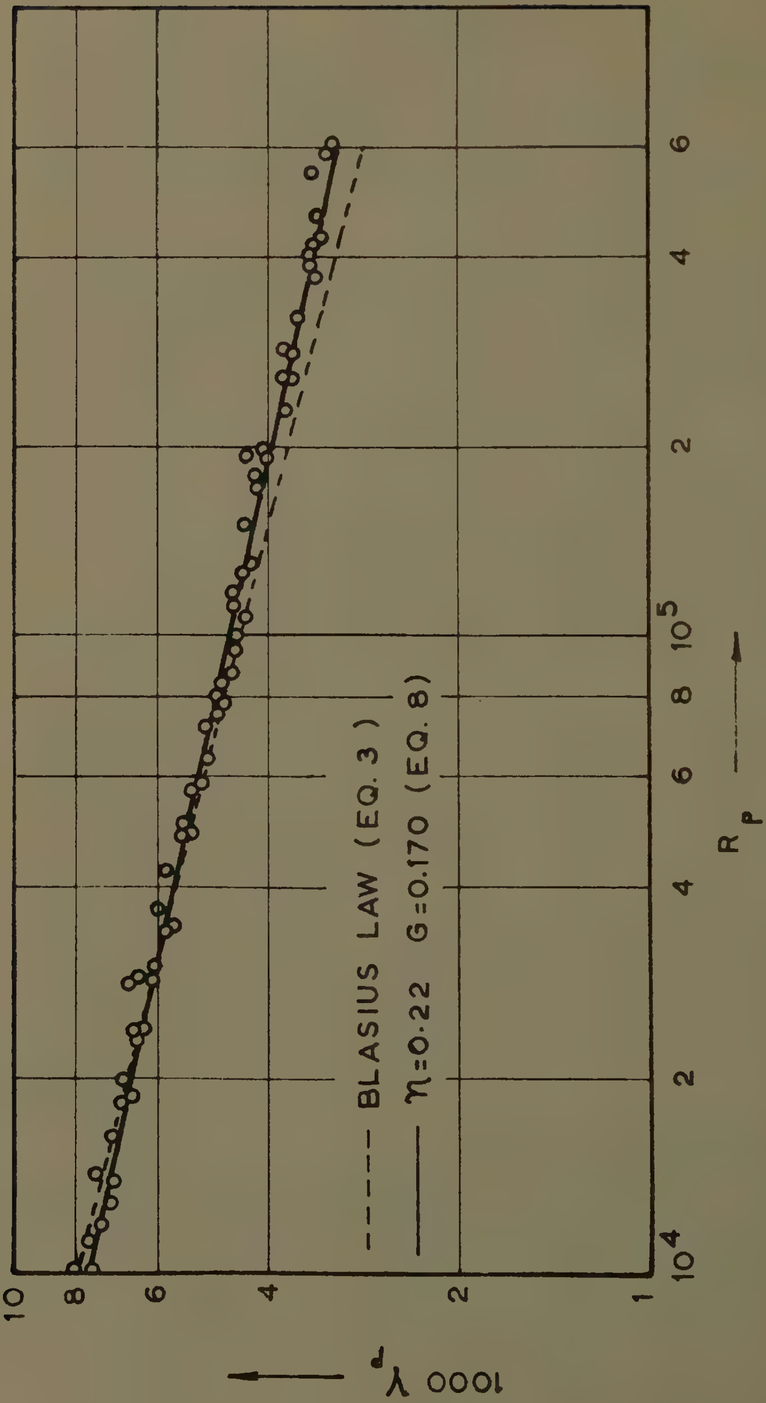

FIG. 1 EMPIRICAL LAWS FOR SKIN FRICTION IN SMOOTH PIPE

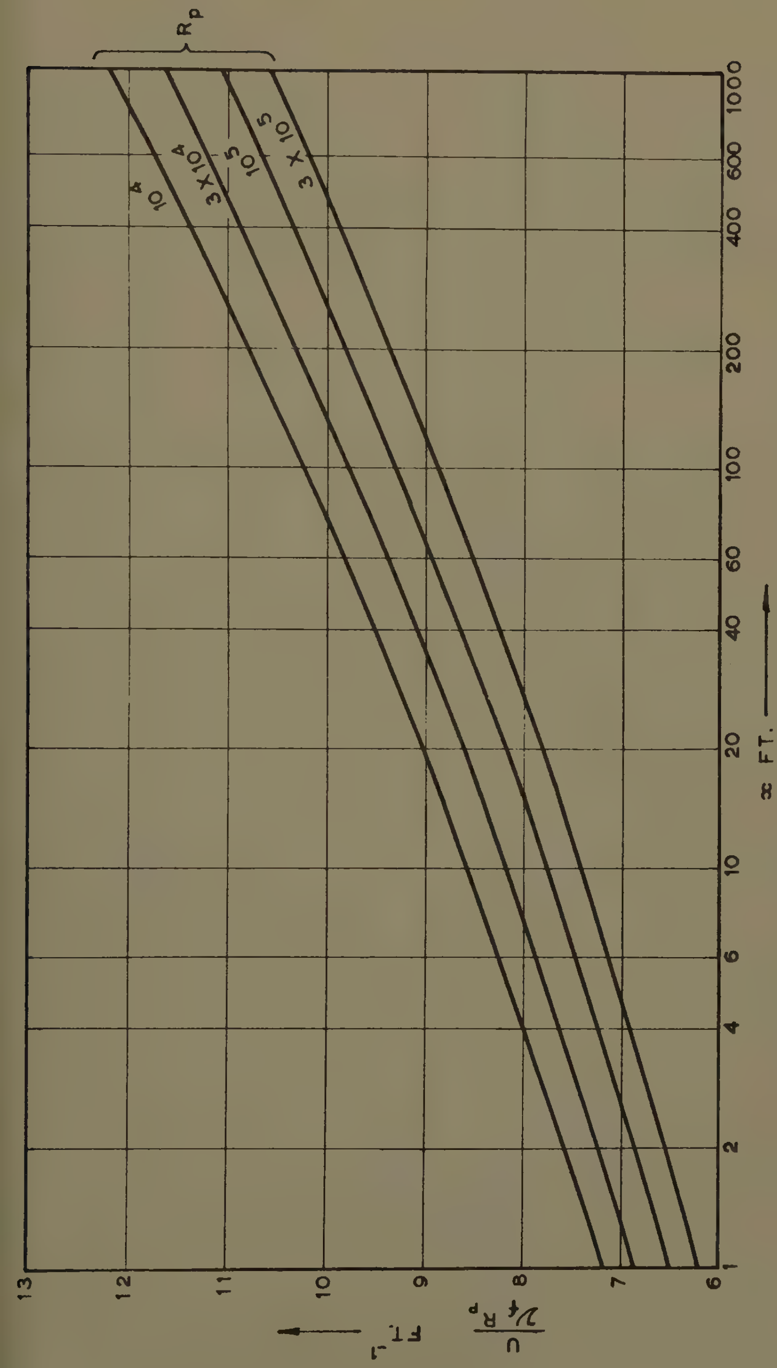

FIG. 2 RELATION BETWEEN CONDITIONS ON FLAT PLATE AND IN PIPE OF DIAMETER 2 IN.

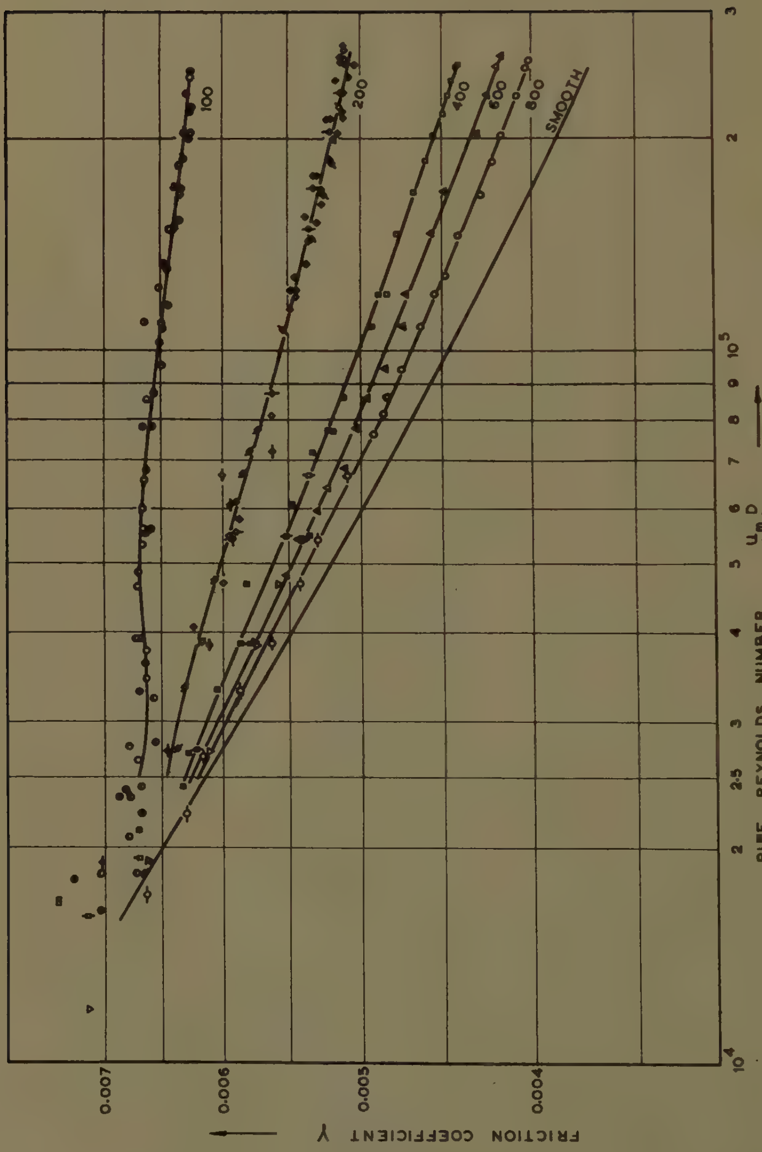

FIG. 3 SKIN FRICTION COEFFICIENTS FOR PIPE WITH PERIPHERAL WIRES

(VALUES OF $\frac{b}{d}$ ARE SHOWN BY NUMBERS ON CURVES)

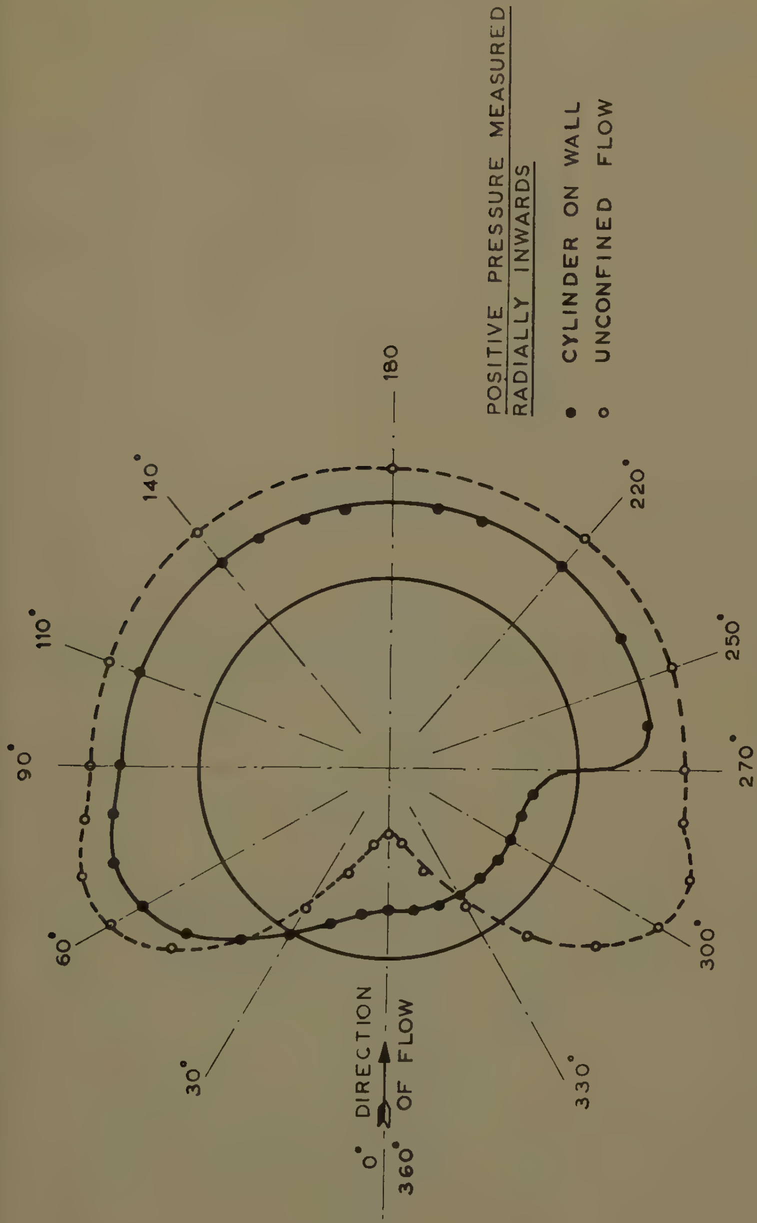

FIG. 4 PRESSURE DISTRIBUTION AROUND CIRCULAR CYLINDER

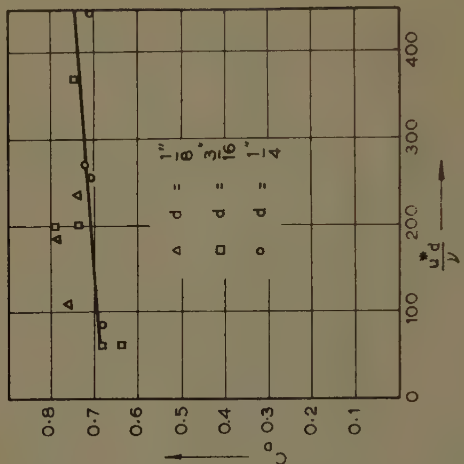

FIG. 5 DRAG COEFFICIENTS FOR CYLINDERS ON A WALL

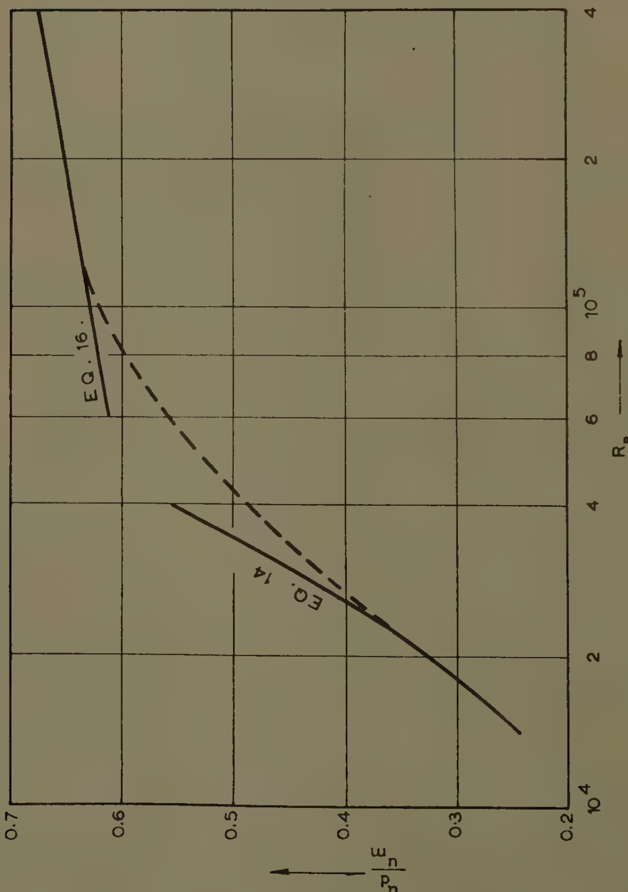

FIG. 6 VARIATION OF $\frac{u_d}{u_m}$ WITH R_p, FOR $\frac{d}{D} = 0.005$

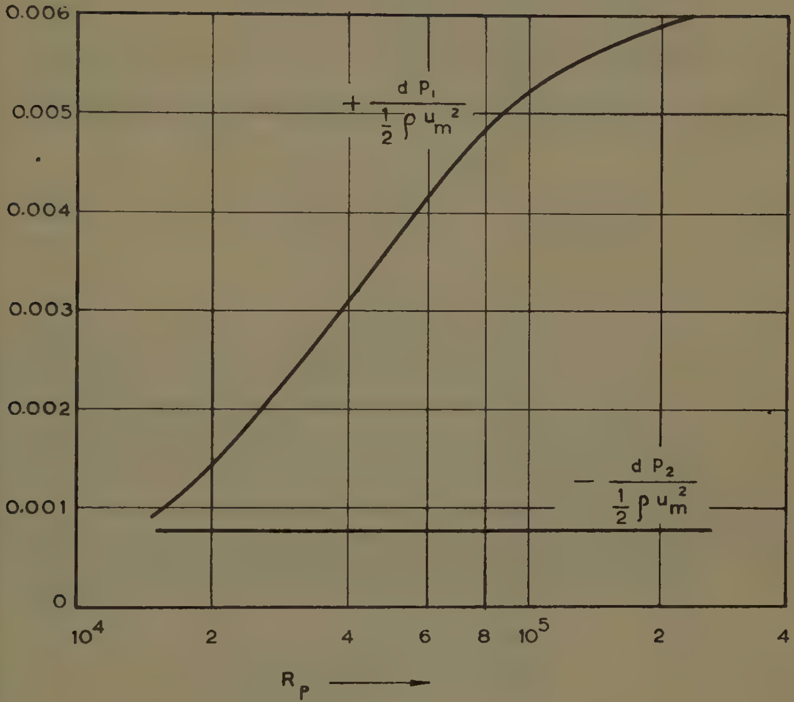

FIG. 7 COMPONENTS OF PRESSURE DROP IN PIPE DUE TO ONE WIRE (d = 0.01 IN., D = 2 IN.)

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THE TURBULENT BOUNDARY LAYER IN A CONICAL DIFFUSER

Harvey R. Fraser[1]
(Proc. Paper 1684)

SYNOPSIS

The development of an axisymmetric boundary layer in a conical diffuser is analyzed and studied in continuation of the work of Uram(1) and Robertson and Holl.(2) A simple, direct method for predicting the behavior of a turbulent boundary layer in a smooth-walled diffuser is proposed. The proposed method is based on the analysis of the boundary layer in two distinct regions as suggested by Ross(3,4) and involves only the solution of algebraic equations. The experimental verification consisted of two tests conducted on a 10° total angle diffuser with different thicknesses of the initial boundary layer. The data from these tests is presented in tabular form and is used to test the applicability of the proposed method for boundary layer calculations.

Symbols and Terminology

The more important symbols and terms are collected and defined below. Other symbols used less extensively will be defined as needed.

c_f	Local wall-shear stress coefficient
D	The outer region shape parameter
L	Prandtl's mixing length in the outer region of boundary layer
R	Radius of conduit or diffuser
r	Radial distance from the center line = R - y
$\mathcal{R}$	Reynolds number, thus $\mathcal{R}_\theta = \dfrac{u_1\theta}{\nu}$
u	Velocity component in the x direction

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u^* Friction Velocity = $\left(\frac{\tau_w}{\rho}\right)^{1/2}$

u_{θ_e} Effective velocity found by extrapolating the law of the wall to $y = \theta$

x, y Axisymmetric coordinates: x is measured from the geometrical start of the diffuser along the diffuser axis; y is measured radially inward from the diffuser wall.

γ Relative effective velocity at momentum thickness = $\frac{u_{\theta_e}}{u_1}$

δ Boundary layer thickness

θ Boundary layer momentum thickness:

Plane or two-dimensional = $\int_0^{\delta} (1 - u/u_1) \frac{u}{u_1} dy$

Axisymmetric or three-dimensional = $\frac{1}{2R} \int_0^{R} (1 - u/u_1) \frac{u}{u_1} d(r^2)$

ν Coefficient of kinematic viscosity

τ Shear stress

τ_{w_e} Effective shear stress at the wall

Subscripts:

e Effective or extrapolated value

i Evaluated at the initial calculation station. For diffuser flow the initial station is the geometrical start of the diffuser

w Evaluated at the wall

θ Evaluated at momentum thickness

1 Evaluated at the outer edge of the boundary layer

2 Value determined using two-dimensional definitions

3 Value determined using three-dimensional definitions

Introduction and Discussion of Two-Dimensional Procedures

The bulk of the literature dealing with boundary-layer flow is concerned with two-dimensional flow without pressure gradients (flat plate) and axisymmetric flows in a small favorable pressure gradient (fully developed pipe flow). A limited amount of effort has been expended in the study of two-dimensional flow in an adverse pressure gradient, but only a few recorded attempts have been made to deal with three-dimensional flows in an adverse pressure gradient. The simplest three-dimensional flow case with an adverse pressure gradient is found in a conical diffuser with axisymmetric flow.

This study, which is a continuation of the work of Uram(1) and Robertson and Holl,(2) resolves some of the difficulties previously reported and suggests a semiempirical method for the prediction of the rate of growth and the approach to separation of the boundary layers considered. The proposed method also allows the prediction of the shape of the velocity profile at any point in the boundary layer.

The method is fundamentally concerned with extending the method proposed by Ross[3,4] for dealing with two-dimensional boundary layers in an adverse pressure gradient. Since the details of the Ross method have been reported by others a brief summary will suffice for this discussion. Ross was one of the promoters of the presently "in vogue" idea of dividing the boundary layer into an inner or wall region and an outer region, each governed by a different flow phenomenon. However, contrary to the popularly held concept that these two regions overlap, Ross found that they were separated by a so-called "blending" region.

In the inner or wall region (exclusive of the laminar sublayer) consisting of approximately 10% of the boundary layer thickness, Ross found that velocity distribution was dependent on the local wall conditions. In this region he applied the so-called "law of the wall" in the following form:

$$\frac{u}{u^*} = 5.6 \log \frac{y u^*}{\nu} + 5.6 \tag{1}$$

which leads to the following expression for the wall shear stress:

$$c_f = \frac{2\gamma^2}{(0.7 + 5 \log \gamma \, R_\theta)^2} \tag{2}$$

Eq. (1) has been shown by Ross, Ludwieg and Tillman,[5] Clauser,[6] and Coles[7] to be independent of the pressure gradient.

In the outer region, which Ross found to be composed of approximately 75% of the boundary layer thickness, the velocity distribution was independent of the local wall conditions and was found to depend in some manner on the conditions upstream, i.e., on the spatial history of the flow. In this outer region Ross found that the following simple expression (which he called the three-halves power velocity deficiency law) gave an excellent fit to most velocity profiles:

$$1 - \frac{u}{u_1} = D\left(1 - \frac{y}{\delta}\right)^{\frac{3}{2}} \tag{3}$$

where

$$D = \frac{2}{3}\,\frac{\delta}{L}\sqrt{\frac{\tau_{w_e}}{\rho u_1^2}} \tag{4}$$

Here D is a shape parameter characterizing the flow in the outer region and hence depends only on the spatial history. It should be noted that the defining equation for D involves the controversial mixing length which has often been maligned but never discarded for the solution of engineering problems. Coles[7] has recently proposed a different but more complicated equation called the "law of the wake" for application in the outer region.

In the blending region consisting of about 15% of the boundary layer thickness Ross found that the velocity distribution was dependent on both the local wall conditions and on the history of the flow. In fact the flow in this region is so complex that he resorted to an empirical relationship between the inner and outer velocity distributions[1,3] and proposed that in drawing velocity profiles the inner and outer regions be joined by a smooth curve. The empirical relationship was expressed by means of the following equation relating γ, a function characteristic of the inner region and used in (2) and D, the outer region shape parameter:

$$\gamma = 0.90\,(1 - 0.68\,D) \tag{5}$$

For flows in a small pressure gradient the momentum thickness development can be determined from the von Karman momentum integral equation. For large pressure gradients the following equation suggested by Ross and Robertson[8] was found to give excellent results for plane two-dimensional flows:

$$\frac{\theta}{\theta_i} = \left[\frac{u_{1_i}}{u_1}\right]^{(2+G)}$$

where the exponent (2 + G) is dependent on the initial value of the momentum thickness Reynolds number $\mathcal{R}_{\theta i}$ as shown in Fig. 1.

Only the longitudinal variation of D needs to be determined to allow boundary layer computations. Ross found that this variation could be expressed as follows:

$$\frac{\delta}{D}\,\frac{u_{1_i}}{u_1} = Ax + \frac{\delta_i}{D_i} \tag{6}$$

where

$$A = 0.025\,(1 + 12\sqrt{c_{f_i}})$$

and x is measured from the beginning of the pressure gradient.

It should be pointed out that the flow parameter $\frac{\delta}{D}\,\frac{u_{1i}}{u_1}$ is strongly influenced by the initial values of δ, D and c_f which are in turn dependent on the spatial history of the flow.

Extension to Axisymmetric Flows

Townsend,[9] Coles[7] and others have found that the division of the boundary layer into an outer and an inner or wall region each governed by a different mechanism is applicable to axisymmetric as well as to two-dimensional flows. Using data from a 7-1/2° diffuser Uram[1] and Robertson and Holl[2] found that the same outer and inner boundary layer regions defined by Ross for the two-dimensional case were applicable to axisymmetric flows. From a mean flow analysis probably the most fundamental difference between axisymmetric and flat plate flow is the fact that the surfaces of constant mean velocity are laterally curved in the axisymmetric case. One would expect this curvature to affect the flow and to manifest itself in at least one or more of the fundamental equations. Another difference noted by Uram[1] and verified by Robertson and Holl is best stated in Uram's words: "Axisymmetric flows can only be meaningfully and physically represented by three-dimensionally defined quantities." This means that in the axisymmetric case the three-dimensional definition must be used for θ and hence for $\mathcal{R}_\theta$ and γ which are defined in terms of θ. Unless otherwise noted, three-dimensional definitions will be used in this discussion.

Coles[7] and others have reliably established the applicability of the law of the wall to the inner or wall region of axisymmetric flows. Coles states, "The relationship is taken for practical purposes as a unique and universal similarity law for every turbulent flow past a smooth surface."

A three-halves power velocity deficiency law has been derived in cylindrical coordinates in reference (10). If y is measured radially inward from the wall of the pipe or diffuser then the equation is identical to the two-dimensional form (3). Uram and Robertson and Holl have offered experimental evidence of its applicability to the outer turbulent region of axysymmetric flows. The derivation of (3) in cylindrical coordinates involves the definition of the shear stress in terms of the mixing length and the assumption that at any cross section the mixing length is constant across the outer region of the boundary layer. The literature is replete with discussions on the applicability of the mixing length approach to three-dimensional axisymmetric flows and, as summarized in reference (10), it is significant to note that most researchers found correlation between theory and experiment very good in the region corresponding to the outer turbulent region. Where serious lack of correlation occurred it was confined to that area near a boundary—the inner or wall region.

The determination of the momentum thickness in a conical diffuser is facilitated by the following simplified equation developed by Ross and Robertson:[8]

$$\frac{\theta R}{\theta_i R_i} = \left[\frac{u_{1_i}}{u_1}\right]^{(2 + G)} \tag{7}$$

where if the three-dimensional value of $\mathcal{R}_{\theta_i}$ is used the exponent (2 + G) is identical in value and varies in the same manner as for two-dimensional flows. The applicability of (7) has been demonstrated in (1) and (2).

Robertson and Holl[2] found that the flow parameter, $\frac{\delta}{D}\frac{u_{1_i}}{u_1}$ (having a dimension of length) used in the two-dimensional theory (6) is not a suitable parameter for the outer region in axisymmetric flow. They did discover, however, that if the above parameter were multiplied by R/R_i, the ratio of the local radius to the initial diffuser radius, then the new parameter thus formed (also having a dimension of length) seemed to correlate all the data taken with the same initial flow conditions. The authors stated that the reason the new parameter tended to correlate their data was not evident.

From a dimensional viewpoint it is felt that if a flow parameter with a dimension of length if characteristic of two-dimensional flow then a flow parameter of length squared should be characteristic of the outer region in three-dimensional flow. Ross[4] supports this type of reasoning when he states in discussing the possible extension of his theory to axisymmetric flow that the various integrated thicknesses would have to be replaced by corresponding areas.

It has previously been indicated that the lateral curvature of the surfaces of constant mean velocity is probably the most significant fundamental difference between two-dimensional flow and three-dimensional axisymmetric flow in a circular conduit. Since R is a manifestation of this curvature, it was reasoned that if the two-dimensional parameter were multiplied by R then a new outer flow parameter is created which has the dimension of length squared and could be characteristic of the outer turbulent region in axisymmetric flows. Since R_i was a constant for the tests previously discussed, the correlation noted by Robertson and Holl will be unaltered by the use of the new parameter except for a change of scale. As demonstrated in Fig. 2 with Uram's data, a variation of the following form is indicated:

$$\frac{\delta}{D}\frac{u_{l_i}}{u_l}R = Ax^2 + \left(\frac{\delta R}{D}\right)_i \tag{8}$$

As in the two-dimensional case it was expected that A should depend on the initial value of the wall shear stress. Using the available axisymmetric data it was found that the following empirical relationship provided a reasonable approximation:

$$A \approx (\sqrt{c_{f_i}} - 0.038) \tag{9}$$

In two-dimensional flow Ross found that $(\delta/\theta)_2$ was a unique function of the parameter D. Experimental data[1,2] does not bear out such a relationship for axisymmetric flow. By a physical and empirical approach[10] it was found in the three-dimensional case that $\frac{\delta}{\theta}$ was a function of θ/R as well as of D. Such a functional variation is subject to proof using some unpublished data,[2] but the algebraic variation was found empirically to be:

$$\frac{\delta}{\theta} - \left(\frac{\delta}{\theta}\right)_2 = 20\frac{\theta}{R}$$

The variation of $(\delta/\theta)_2$ for plane two-dimensional flow with D has been given.[4] When this relationship is substituted into the above equation, a fourth degree equation in D results. It was found that if this fourth degree equation were plotted for various constant values of $\frac{\delta}{D\theta}$ using D and θ/R as variables then a series of linear relationships resulted as shown in Fig. 3. This relationship permits the determination of D at any x once δ/D and θ have been determined.

It has been shown[1,2] that the inner and outer profiles for axisymmetric flows are also related by (5). The applicability of this equation has been further demonstrated with the data collected in this study.

All the necessary relationships are now available to permit boundary layer calculations in a diffuser once the initial values of δ, θ, D, u_1, and c_f are known. It is assumed that the velocity distribution in the core outside of the boundary layer can be computed from the pressure distribution.

Calculation Procedure

The detailed calculation procedure is summarized as follows:

1. Compute $\mathcal{R}_{e_i}$ and read the value of $(2 + G)$ from Fig. 1.
2. Using (7) determine θ at the section under consideration and compute the value of θ/R.
3. Compute the value of the outer flow parameter, $\frac{\delta}{D}\frac{u_{1_i}}{u_1}R$ using (8) and (9). This allows the computation of $\frac{\delta}{D}$ and $\frac{\delta}{D\theta}$.
4. From Fig. 3 determine the applicable value of D using the known values of $\frac{\theta}{R}$ and $\frac{\delta}{D\theta}$ found in steps 2 and 3. The boundary layer thickness can now be determined from the known values of $\frac{\delta}{D}$ and D.
5. Compute γ from (5).

6. Plot velocity profiles if desired by using (1) and (3). In the blending region the velocity profile must be faired between the outer and inner curves.

7. If the local shear stress coefficient is desired, (2) is available.

Experimental Apparatus and Procedures

The empirical relationships referred to above were all based on the data from the 7-1/2° diffuser.[1,2] In order to test the proposed theory a series of two tests (A and B) were run on a 10° diffuser. The open flow circuit, sketched in Fig. 4, consisted of a 24" blower, a reducing section, a 17.4" diameter settling section which contained a honeycomb flow straightener, a nozzle, a variable length of 6" straight pipe and finally a conical diffuser which together with the straight pipe was made of aluminum castings carefully mated and polished in pairs. Provisions were made for piezometer and pitot traverse stations at approximately 2-1/4" intervals along the diffuser. Detailed velocity traverses were made with hypodermic type total head tubes (0.028" diameter stainless steel sting mounted on a 1/8" supporting tube) fitted into a traverser which allowed accurate positioning in 0.001" increments. All pressure readings were made with a micromanometer. Displacement and momentum thickness values were determined with a planimeter from plots of the measured data. Values of D, δ, and c_f were obtained by graphical fitting of curves using procedures described in (10).

Two test arrangements were employed to provide different entrance and exit conditions from the diffuser. In test A the entrance boundary layer was about 0.4" thick with an initial free stream velocity of 171 fps ($/R_{\theta_i} = 3.31 \times 10^3$). In test B the corresponding values were about 0.3" and 189 fps ($/R_{\theta_i} = 2.88 \times 10^3$). In test A an attempt was made to use a diffuser length that would allow the flow to approach but not quite reach separation. In test B the diffuser was lengthened until separation was known to occur. A summary of experimental data for each test is shown in Table 1.

Producing Axisymmetric Flow

The production of axisymmetric flow conditions in the diffuser proved to be a troublesome, time-consuming, and laborious process. A trip wire in the nozzle exit just upstream from the straight pipe section was necessary to insure symmetric transition from laminar to turbulent flow but even with the trip wire the flow was initially asymmetric. Extensive experimentation was accomplished with a series of fine and coarse screens with various combinations of honeycomb flow straighteners in an effort to produce an acceptable condition of symmetry. Finally all screens were removed and one honeycomb, consisting of 1" diameter tubes 6" long carefully cemented in a section of the flow circuit ahead of the entrance to the nozzle, produced reasonably acceptable conditions. The results observed would indicate that screens are of doubtful value in producing truly axisymmetric flow. Baines and Peterson[11] observed that certain fine meshes produced unstable conditions downstream. In test A and B even coarse screens produced detrimental effects. Collar[13] pointed out that there is an optimum screen mesh for producing uniform velocity under any stated condition of flow but that any size screen should reduce the magnitude of the velocity variations.

Separation

Ross[4] found that a value of D = 1.3 ± 0.1 was quite a reliable indicator of separation in two-dimensional flow. As previously indicated, separation occurred near the end of the diffuser in test B where values of D $>$ 1.3 were observed. During trial runs of the apparatus when the flow was asymmetric numerous velocity profiles were collected in the diffuser at places where separation was occurring. In these instances the experimental D values were $\gtrsim$ 1.3. At the extreme end of the diffuser in test A an experimental value of D = 1.3 was noted but various tests indicated that although the flow was approaching separation, it had not quite reached a separated state. Other researchers[1, 2] have recorded D values as high as 1.2 in diffusers where no separation was indicated. Based on the limited data available it would appear that D = 1.3 ± 0.1 is a reasonable criteria for the determination of separation in axisymmetric flows.

Comparison of Experimental and Calculated Values

The method proposed in this paper was applied to tests A and B and the growth and shape of the boundary layer were predicted. To obtain the entrance or initial data the values of D, θ, u_1 and δ found experimentally at two stations (Stations 1 and 2 in Table 1) in the straight pipe upstream from the start of the diffuser were extrapolated to the diffuser entrance. The initial value of c_{f_i} was determined from the following equation suggested by Ross[13] for equilibrium flow in a pipe:

$$\sqrt{\frac{2}{c_{f_i}}} = 6.2 + 5.2 \log \mathbb{R}_{\theta_i} \tag{10}$$

This equation was used since it requires only the initial value of $\mathbb{R}_\theta$. The results obtained are within about 10% of the values of c_{f_i} obtained by the extrapolation procedure described above. A comparison of experimental and calculated values of θ, D and δ for the diffuser flows is graphically presented in Fig. 5. It will be noted that except near the end of diffuser B where separation is known to occur, the correlation between the computed and the experimental is excellent for the momentum and boundary layer thicknesses. The correlation for the outer shape parameter D is not quite as good but at worst the deviation is only about 10%. In addition to the comparisons shown in Fig. 5, two randomly selected sample velocity profiles were calculated and are plotted with the experimental points in Fig. 6.

CONCLUSIONS

Based on the theoretical considerations and on the experimental analysis presented in this paper, it appears that the following conclusions are warranted:

1. The calculation method proposed in this paper is reasonably effective in predicting the shape as well as the rate of growth of the axisymmetric boundary layers considered in this study.
2. A value for the outer shape parameter of D = 1.3 ± 0.1 seems to be a reliable indication of separation in axisymmetric as well as in two-dimensional flows.

3. The practical application of the proposed method is limited by the necessity of determining the initial values of D, δ, and θ by experimental methods.

RECOMMENDATIONS

1. The empirical relationships proposed in this paper should be further tested with additional data taken under different rates of flow and different entrance conditions using diffusers of different size and different central angles. Special attention should be given to relatively thicker boundary layers, i.e., $\frac{\delta}{R} \to 1$. It is quite possible that as the boundary layer thickens and the flow approaches the fully developed state various forces or second order effects, insignificant in relatively thin boundary layers, might become important. As pointed out by Ross[4] and the author in reference (10) a minimum of 15 to 20 points are required on each velocity profile for the proper determination of D and δ.
2. The possibility of determining the initial values of D, δ, and θ without recourse to velocity traverses should be examined. This determination is essentially a problem of the growth of a boundary layer in the entrance region of a pipe. Ross and McGinley[14] and Ross[15] have proposed workable relationships for the determination of some of these quantities if the effective origin of the boundary layer can be determined.

ACKNOWLEDGEMENT

This paper is based on a thesis submitted to the Graduate College at the University of Illinois in July 1956 in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Theoretical and Applied Mechanics. The author is especially grateful to Dr. James M. Robertson for his guidance during this investigation. The nozzle and diffuser for experimental work were kindly loaned by the Bureau of Ordnance, U. S. Navy, from the Ordnance Research Laboratory at the Pennsylvania State University.

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Sta. No.	x in	R in	Q cfs	u_1 fps	$c_f \times 10^{-3}$	D	δ in	δ_2^* in	δ_3^* in	θ_2 in	θ_3 in	γ
TEST A												
1	- 2.20	3.032	32.9	170	3.50	0.285	0.38	0.055	0.053	0.039	0.037	0.700
2	- 0.57	3.042	33.6	171	3.50	0.300	0.40	0.058	0.056	0.041	0.039	0.702
3	1.49	3.152	33.4	161	2.70	0.350	0.49	0.078	0.075	0.054	0.051	0.650
4	2.51	3.240	33.3	153	2.00	0.400	0.53	0.100	0.098	0.066	0.062	0.580
5	5.01	3.459	32.9	138	1.40	0.620	0.61	0.162	0.150	0.092	0.088	0.495
6	8.21	3.739	33.3	126	0.80	0.800	0.84	0.268	0.251	0.138	0.129	0.360
7	10.46	3.936	33.4	120	0.58	0.925	1.02	0.373	0.347	0.173	0.156	0.300
8	11.64	4.040	33.7	117	0.52	0.975	1.12	0.412	0.384	0.190	0.170	0.290
9	13.78	4.226	33.2	113	0.35	1.15	1.38	0.556	0.507	0.223	0.196	0.230
10	15.94	4.416	33.8	110	0.25	1.20	1.62	0.694	0.612	0.265	0.222	0.190
End	18.48	4.638	33.0	106	0.16	1.30	1.90	0.857	0.746	0.306	0.251	0.160
TEST B												
1	- 2.20	3.032	36.6	188	3.50	0.260	0.31	0.044	0.043	0.029	0.028	0.695
2	- 0.57	3.042	37.1	189	3.60	0.275	0.32	0.045	0.044	0.031	0.030	0.742
3	1.23	3.128	36.8	179	2.90	0.325	0.36	0.059	0.057	0.040	0.038	0.664
4	2.51	3.240	36.3	167	2.20	0.425	0.40	0.082	0.081	0.053	0.051	0.580
5	5.01	3.459	36.6	151	1.50	0.625	0.51	0.134	0.128	0.081	0.076	0.490
6	8.21	3.739	37.1	138	1.00	0.750	0.74	0.228	0.217	0.122	0.113	0.402
7	10.46	3.936	36.7	128	0.78	0.880	0.93	0.320	0.298	0.157	0.142	0.375
9	13.78	4.226	37.0	121	0.40	1.050	1.30	0.513	0.461	0.219	0.190	0.260
10	16.04	4.425	36.9	117	0.30	1.180	1.55	0.663	0.577	0.263	0.221	0.200
11	20.12	4.782	36.8	111	0.16	1.250	2.10	0.938	0.806	0.334	0.271	0.140
12	22.46	4.985	35.9	108	0.08	1.300	2.45	1.165	0.969	0.383	0.295	0.080
End	24.07	5.122	34.7	105	0.00	1.380	2.70	1.300	1.085	0.409	0.308	0.000

Q = Flow rate in cfs δ_2^* and δ_3^* = Two and three-dimensional displacement thicknesses

TABLE I SUMMARY OF EXPERIMENTAL DATA

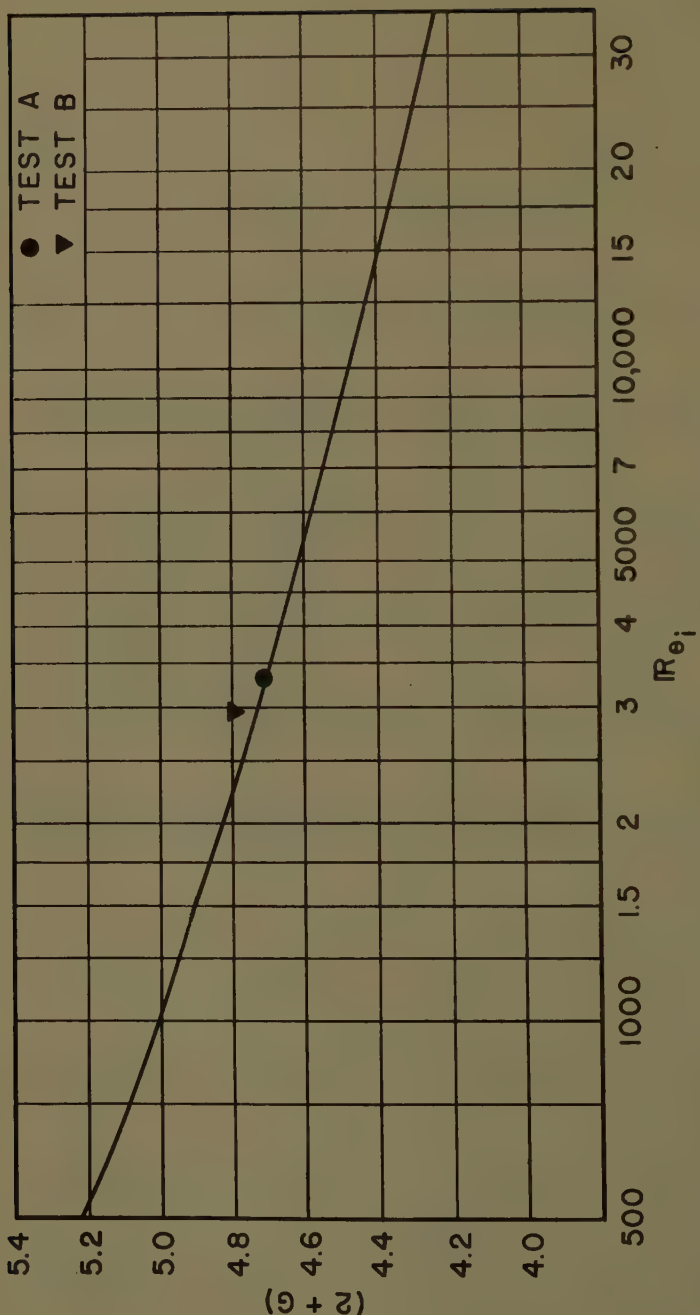


FIG. I VARIATION OF (2 + G) WITH INITIAL REYNOLDS NO.

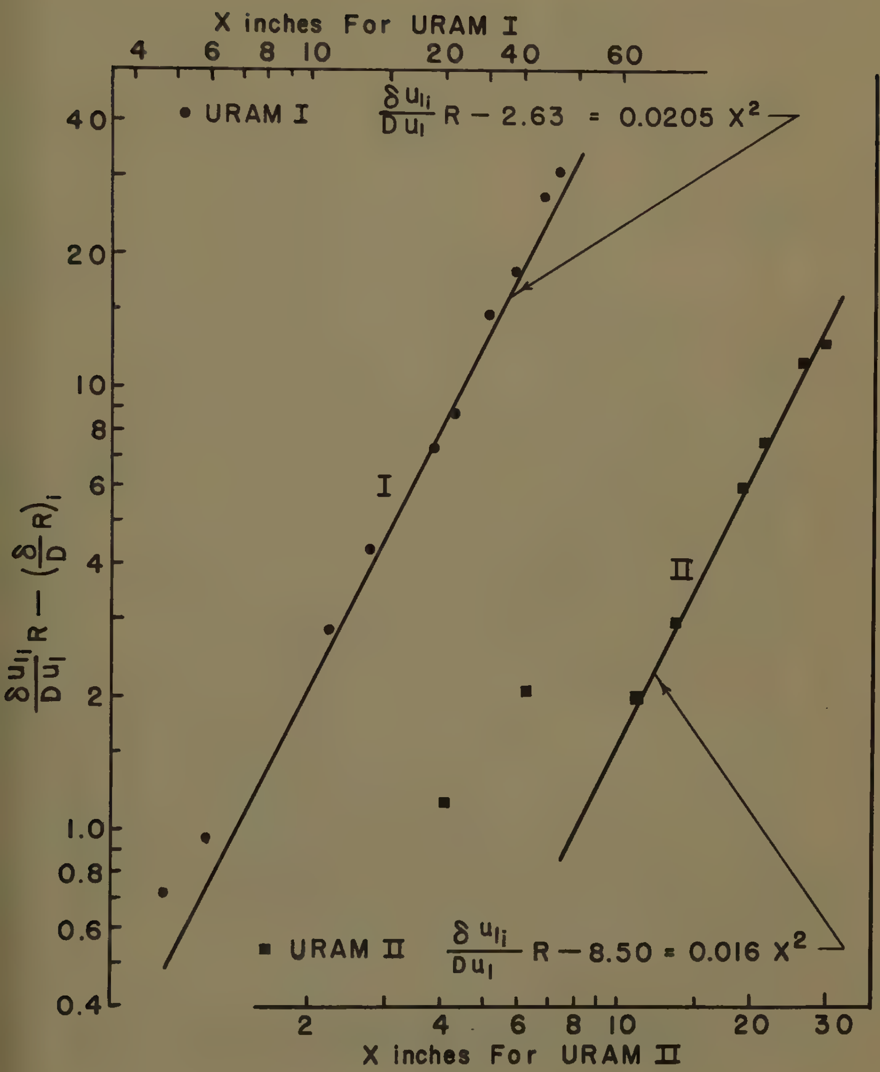


FIG. 2
VARIATION OF THE FLOW PARAMETER WITH DISTANCE FOR URAM'S DATA (1)

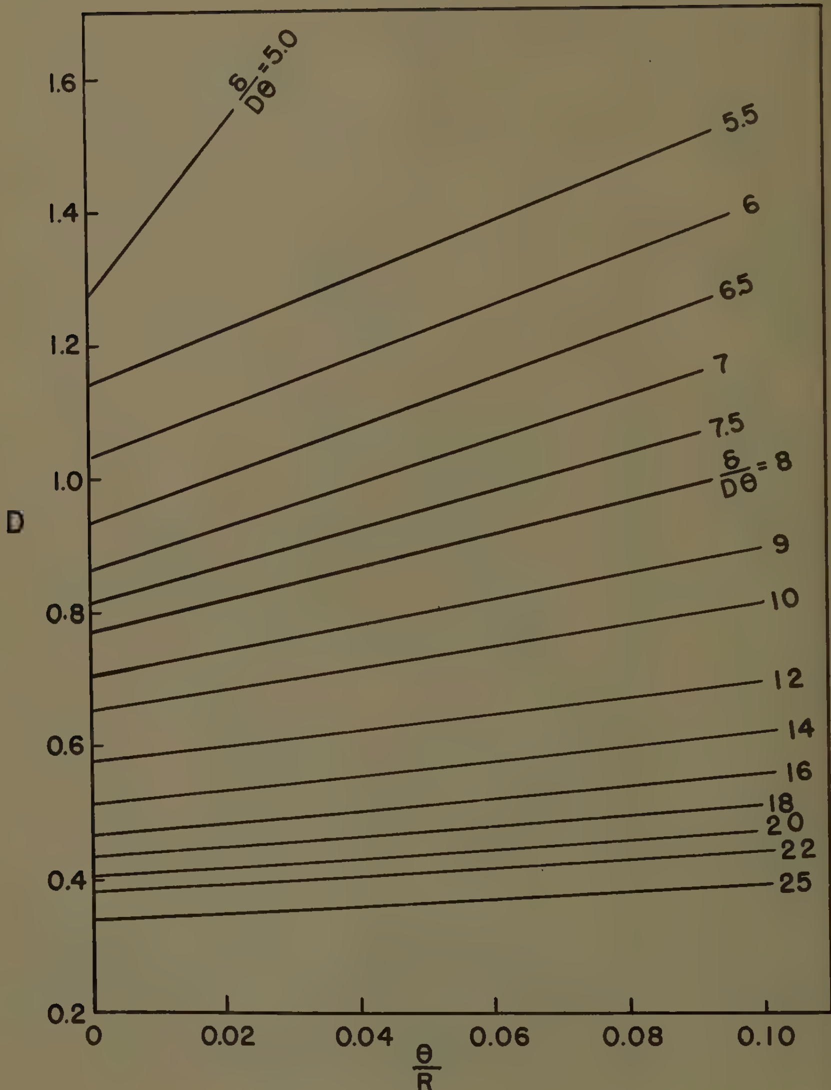
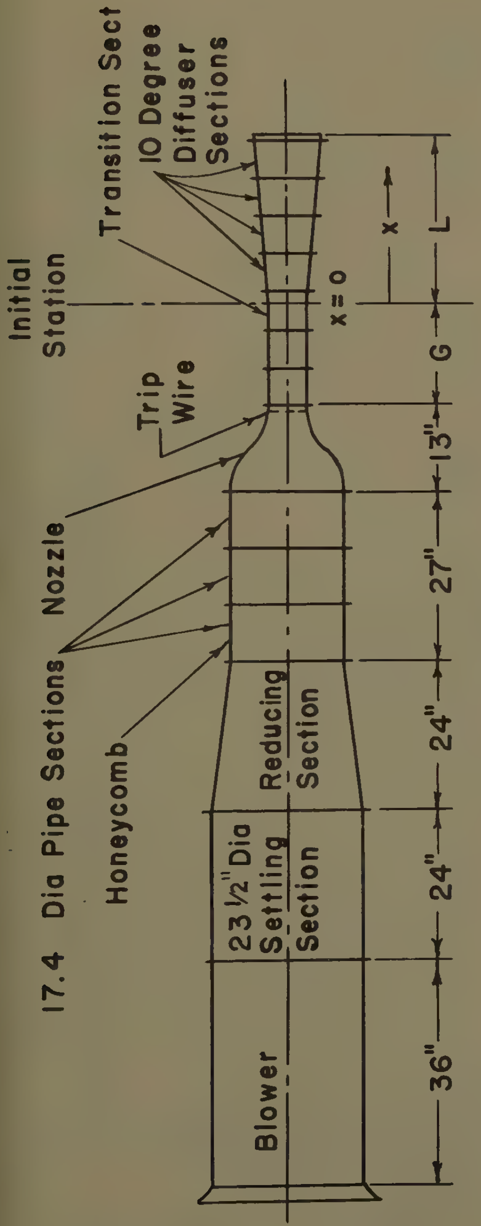


FIG. 3

VARIATION OF THE OUTER REGION SHAPE PARAMETER D, AS A FUNCTION OF $\frac{\theta}{R}$ AND $\frac{\delta}{D\theta}$



LEGEND

G – Variable Entrance Pipe Length – 6.06" Dia

x – Longitudinal Distance From Initial Station

L – Total Diffuser Length

G = 15.8" For Test A

L = 18.82" For Test A

G = 9.8" For Test B

L = 24.40" For Test B

FIG.4 LINE DRAWING OF FLOW CIRCUIT

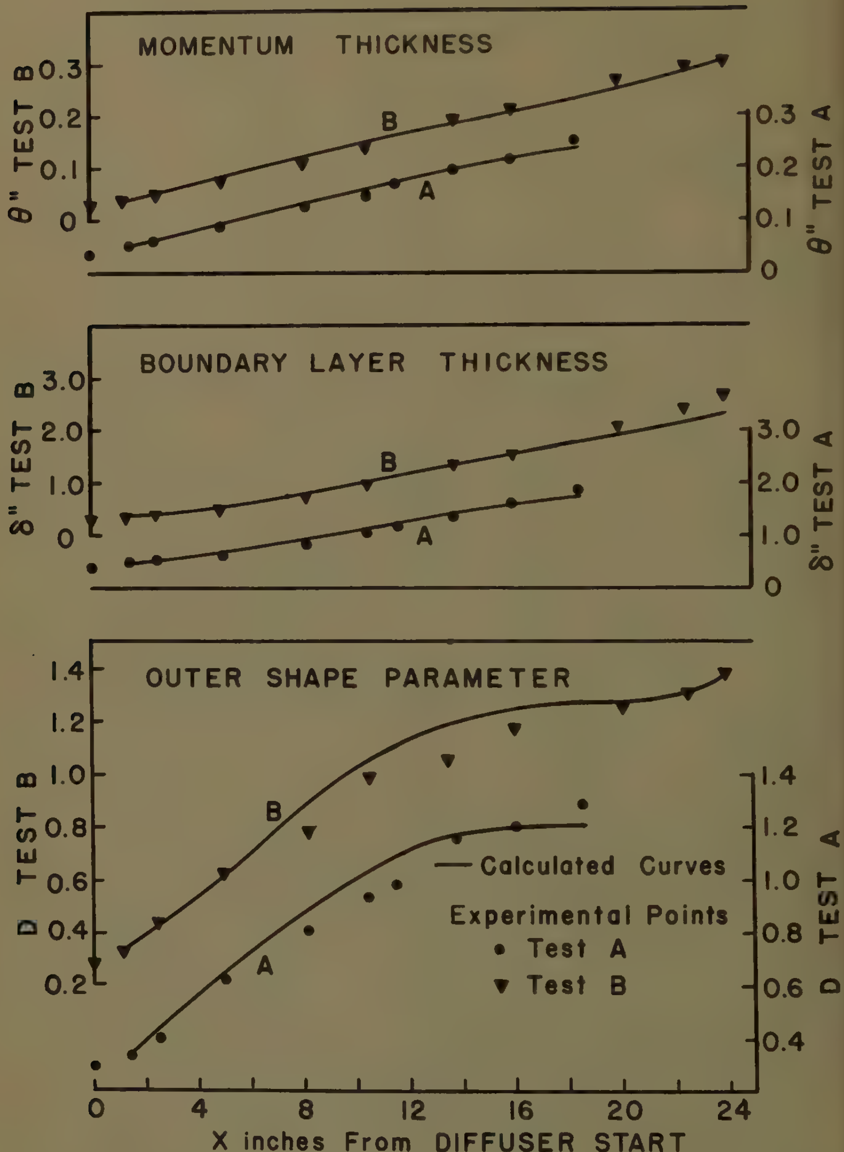


FIG. 5
COMPARISON OF VARIOUS CALCULATED AND EXPERIMENTAL VALUES

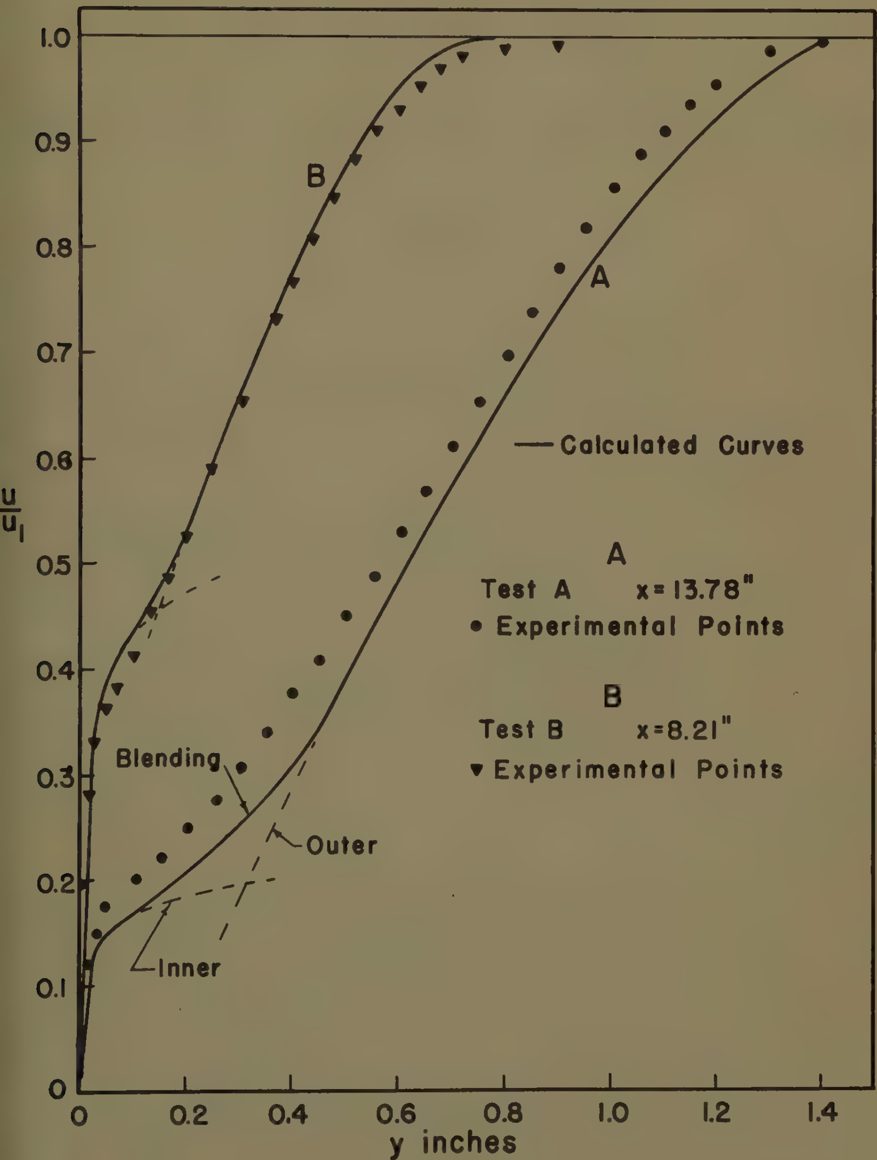


FIG.6 SAMPLE CALCULATED AND EXPERIMENTAL VELOCITY PROFILES

Journal of the

HYDRAULICS DIVISION

Proceedings of the American Society of Civil Engineers

CONTENTS

DISCUSSION
(Proc. Paper 1690)

(Over)

THE HYDRAULIC JUMP AT AN ABRUPT DROP[a]

Discussions by M. B. McPherson and J. W. Forster

M. B. McPHERSON,[1] A. M. ASCE.—The most valuable contribution by the authors is their presentation of pressure and velocity measurements. In addition, their studies have extended appreciably the range of performance previously reported. The objectives of this discussion are to compare and integrate their findings with results published recently by others. Apparently the authors have overlooked the data published for a $\Delta Zo/Y_1$ of 6.67 for F_1 from 1 to 4,(1) and for an F_1 of one for $\Delta Zo/Y_1$ from 2 to 6.67.(2)

The curves of Fig. D-1 include the upper limit of "Jump B" (or "plunging nappe" in references (1) and (2)), from Figs. 6, 7 and 8. These curves as projected agree with the points from reference (2) for a falling tailwater at F_1 equals one (circled points). The statement by the authors under "Conclusion-2: As the relative downstream depth is reduced from its upper limit, . . ." suggests that all their data may have been obtained only with a falling tailwater. However, if the lower limits of the "Wave" regime were extended they would practically intersect the "rising tailwater" points. According to the distribution of the points in Fig. D-1, the effect of tailwater direction appears to become more pronounced as $\Delta Zo/Y_1$ increases. The curve for $\Delta Zo/Y_1$ of 4 does not seem to be consistent with the other curves for values of 2 and 3 at the higher range of F_1. The delineation of the upper limit of "Jump B" is very important since most of the high bottom velocities were observed for this case (Figs. 9, 10 and 11).

The "Jump A" limits (a "Wave" is called a "riding nappe" and "Jump A" is called "hydraulic jump" or a "small roller" in references (1) and (2)), reproduced in Fig. D-2 are generally consistent. The reference data points for an F_1 of one indicate a small difference between a "rising" and a "falling tailwater" for this case. Is it possible that the pairs of curves for "Jump A" are for these two tailwater conditions? Presumably equivalent data by Mr. Hsu(4,5) differs from Fig. D-2 in that Hsu noted a sudden drop in Y_2/Y_1 for:

$\Delta Zo/Y_1$	at F_1
1	4
2	5
3	6
4	8

from data points which (up to the above F_1) closely satisfied Eq. (4). Comparing the limits given by the authors and the trace of Eq. (4) in Fig. D-2 there

a. Proc. Paper 1449, December, 1957, by Walter L. Moore and Carl W. Morgan.

1. Research Engr., Philadelphia Water Dept., Philadelphia, Pa.

is evidently considerable disagreement between their data and the corresponding data by Mr. Hsu. Beyond the values of F_1 given in the above table, the lowered points satisfied Eq. (5). To the contrary, the authors' data and the data from reference (1) are in agreement with regard to trend.

All measurements of h_D/Y_1 plotted in Figs. 3, 4 and 5 have been approximately reproduced in Fig. D-3. The data by Mr. Hsu, beyond the sudden drop in values described above, satisfied Eq. (5). Note that the arbitrarily drawn curves of fit (solid lines) seem to approach $h_D = Y_2$ as a limit. Fig. D-2 shows that Eq. (4)(for $h_D = Y_2$) is most closely satisfied at F_1 equals one; the trend of the arbitrarily drawn F_1 curves in Fig. D-3 bear this out. Fig. D-3 indicates a satisfactory correlation and consistency in the measurements reported by the authors.

It is interesting to note in passing that the lower limits for "Jump B" given in Figs. 6, 7 and 8 ("sweepout" condition) do not differ appreciably from the values of Y_2/Y_1 for a jump on a horizontal floor ($\Delta Zo/Y_1 = 0$):

F_1	Y_2/Y_1
2	2.4
4	5.2
6	8.0
8	10.8

The erosive potential of "Jump B" has been very clearly underscored by the authors, through their presentation of measured velocities. Unfortunately, the relation between the curves of Figs. 9, 10 and 11 and the curves of Figs. 6, 7 and 8, in terms of Y_2/Y_1, are not stated; only the jump type and F_1 are indicated in the former figures, in an abbreviated form.

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J. W. FORSTER,[1] A. M., ASCE.—The authors have made a systematic investigation of the effect of the position of the hydraulic jump relative to an abrupt drop on the form of the jump and on the effectiveness of the stilling action. This comprises a valuable extension of the study presented by Mr. Hsu(5) and provides data useful for design.

In a typical problem, the upstream and downstream depths for a given discharge (hence F_1 and Y_2/Y_1) are known, and the size of sill in the channel bottom required to stabilize a hydraulic jump is desired. When the downstream depth is greater than the sequent depth for a normal jump, there is, of course, no danger of the jump being swept downstream. Rather, it will tend to move upstream against the face of the spillway or sluice gate and be drowned. To prevent such drowning, with associated high bottom velocities,

1. Supt. of Hydraulic and Power Studies, Aluminum Co. of Canada, Ltd., Kitimat, B. C., Canada.

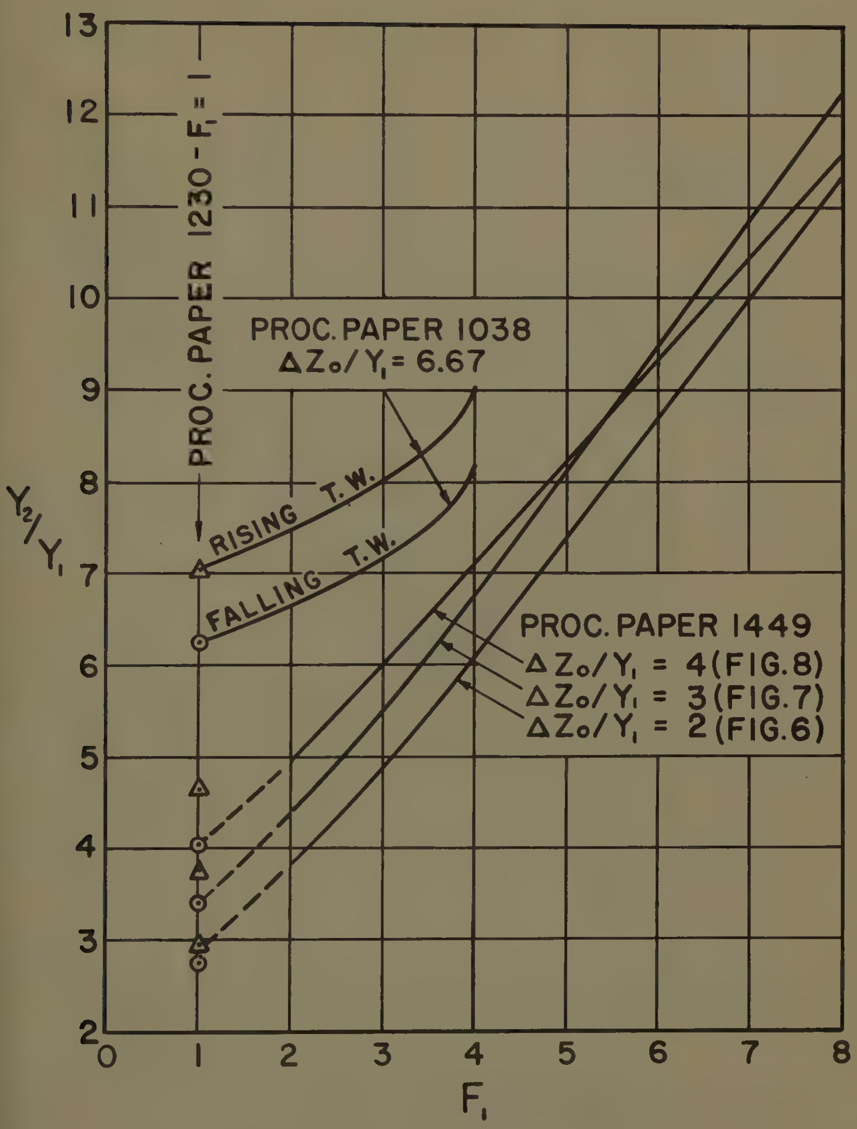

FIG. D-1: UPPER LIMIT OF "JUMP B"

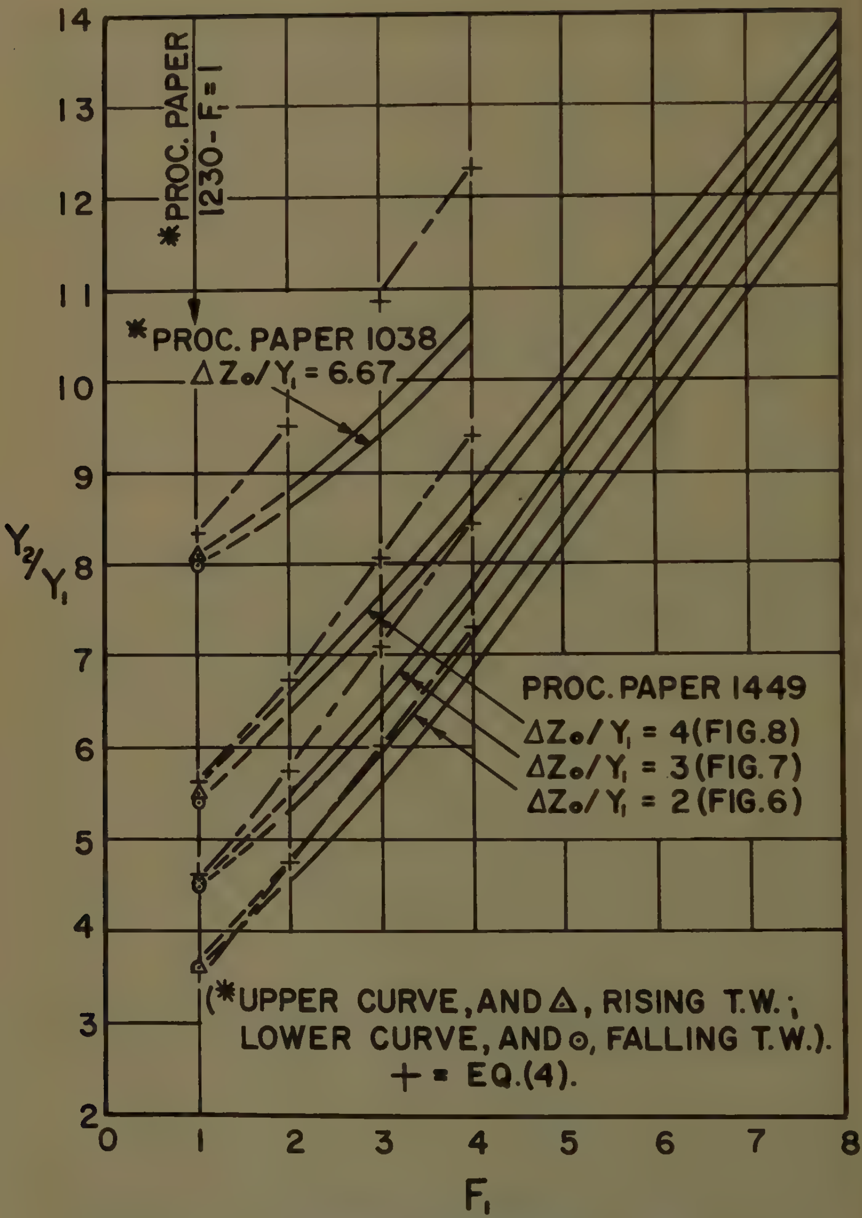

FIG. D-2: "JUMP A" LIMITS

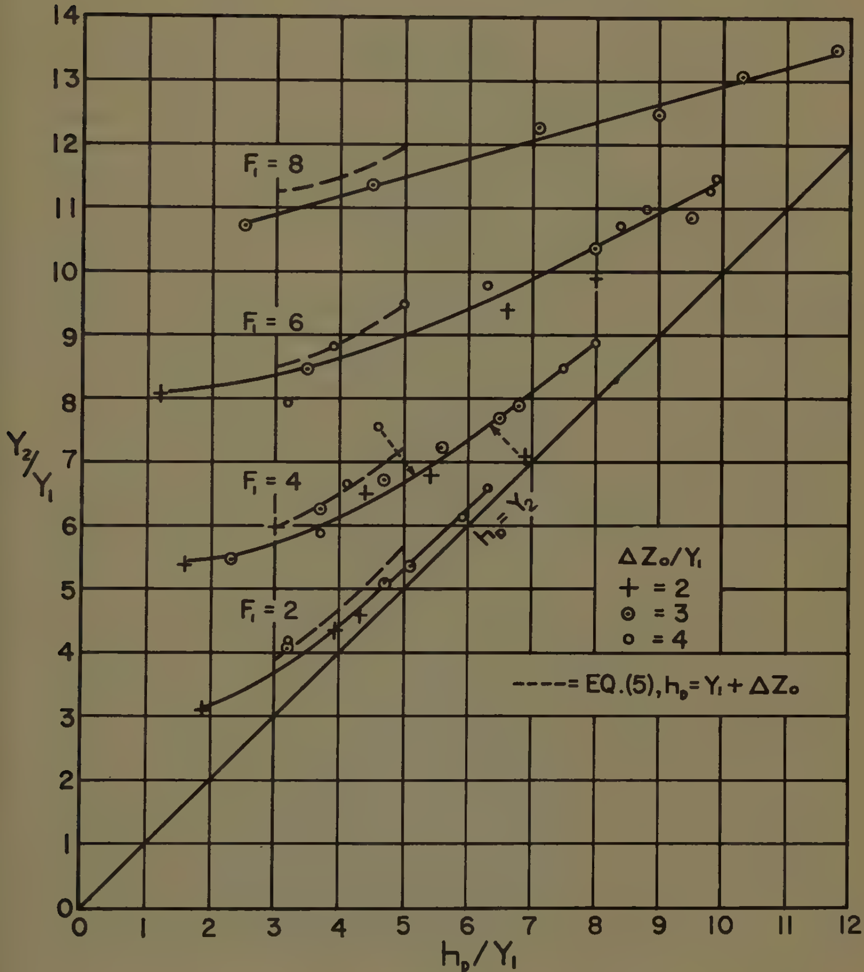

FIG. D-3: COMPARISON,
PIEZOMETRIC HEAD MEASUREMENTS

a drop in the channel bottom is required. Information provided by the authors show, for any given upstream and downstream flow conditions, how the form of the jump and its effectiveness in reducing bottom velocities vary with the size of drop.

For example, with $F_1 = 4$ and $Y_2/Y_1 = 7$, Figs. 6 and 9 show that introducing a drop twice the upstream depth Y_1 will yield a type A jump with a downstream bottom velocity $V_b = -1.5V_2$; very effective stilling. If the drop is increased to $3Y_1$, Figs. 7 and 10 show that a wave type jump will form with $V_b = -2V_2$; still quite effective stilling. If the drop is further increased to $4Y_1$, Figs. 8 and 11 show that a type B jump will form, with V_b increased to $5V_2$; comparatively ineffective stilling. An interesting question, in this latter case, is whether stilling would be as effective if no drop at all in the channel were introduced, and the jump were drowned.

An approximate answer to this question can be obtained from information in Mr. Henry's discussion of the paper "Diffusion of Submerged Jets", Transactions ASCE, p. 639, (1950). Mr. Henry investigated conditions within a drowned jump for a Froude number of 2.0 at the gate outlet (see his Fig. 33). Assuming a coefficient of contraction of 0.8 for the rounded gate lip, this case would correspond to $F_1 = 2.8$ and $Y_2/Y_1 = 5$. The velocity distribution measurements made by Mr. Henry indicate, at a distance below the vena contracta of $L = 1.5Y_2$, that the bottom velocity $V_b = 6V_2$.

As an alternative to this drowned jump, Mr. Moore's Fig. 6 shows that for the same upstream conditions ($F_1 = 2.8$ and $Y_2/Y_1 = 5$), by introducing a drop $\Delta Z_o = 2Y_1$, a wave type jump can be produced. At $L = 1.5Y_2$, Fig. 9 shows that $V_b = 1.7V_2$. This is much better stilling than in the case of the drowned jump.

On the other hand, if the depth of drop were increased to $\Delta Z_o = 3Y_1$, Fig. 7 indicates that a type B jump would form, with the toe downstream of the drop. Fig. 10 shows that the downstream bottom velocity would then reach a value $V_b = 5V_2$: almost as great as the $6V_2$ value indicated by Mr. Henry's data for the case of a drowned jump with no drop.

Where the downstream depth is less than the sequent depth for a normal jump, a rise, rather than a drop, in the channel bottom is required to prevent the jump from being washed downstream. It should be noted that whereas an over-dimensioned rise will result in a drowned jump with any high velocity bottom currents directed upward and away from the bottom by the sill, an over-dimensioned drop will cause the jump to recede downstream with resulting high bottom velocities. Accordingly, to ensure downstream protection, it is safer for the designer to keep the size of a rise somewhat greater and the size of a drop less than the optimum dimension computed according to the design charts.

In the case of the abrupt rise also, the force on the face of the sill and the effectiveness of the stilling vary with the position of the jump relative to the rise. This case was investigated by Forster and Skrinde(3) for one relative position of the jump; their investigation could be extended to include a range of relative jump positions in the same way as the authors have done in the case of the abrupt drop.

Since the authors were concerned only with the condition of jump formation, they did not investigate values of Y_2/Y_1 less than 1. This range, corresponding to super-critical flow shooting over an abrupt drop and continuing downstream, would comprise a nice extension, not only of the authors' present study, but also of Mr. Moore's(2) previous investigation of the free overfall.

A theoretical plot based on an analysis by Merit P. White* has been presented,** but this range has not been investigated experimentally. It appears to be a logical next step in the completion of the overall picture of flow over sills.

*See Reference 2, p. 1361.
**See Reference 3, p. 1019, Fig. 23, Region VI.

FLOOD FREQUENCIES DERIVED FROM RAINFALL DATA[a]

Discussion by H. C. Riggs and Manuel A. Benson

H. C. RIGGS,[1] A. M. ASCE and MANUEL A. BENSON,[2] A. M. ASCE.—The authors describe procedures for synthesizing flood peaks from rainfall records. They evaluate the procedures by comparing synthesized and known peaks and then use the developed relations to extend the length of record of peaks at the station. Their methods require an annual flood peak record of about 15 years and rainfall records (including at least one recording record) substantially longer than the record of annual flood peaks. Procedures involve (1) distributing daily rainfall totals among portions of a day according to the known distribution at a recording rainfall station, (2) a search for suitable volume rainfall-runoff relationships already developed or the development of such relationships, and (3) the development of unit hydrographs. Much of this work must be done for each station whose flood record is to be extended. In the writers' opinion these procedures do not lead to more reliable flood-frequency relations despite the prodigious amount of labor involved.

The accuracy of the results were evaluated by showing that synthesized and observed floods for Codorus Creek at Spring Grove, Pa., define approximately the same frequency curve for the period 1933-54. It is significant that individual flood peaks were not accurately duplicated. Close correspondence between known and synthesized flood peaks is attained only by comparing floods of equal rank. Such correspondence is not a measure of the reliability of the synthesized floods but is due to basic principles involved in the ranking procedure.

If two random equal-sized sets of data with approximately the same distribution and range are arranged in order of magnitude, the corresponding items will agree very closely. Maximum rainfall was used to estimate maximum discharge. Both have been shown to approximate extreme-value distributions. The range of the synthesized set of flood peaks is brought into close agreement with the observed range by the adjustment curve of the authors' Fig. 6. Therefore, conditions necessary for duplicating the ranked observed floods are fulfilled. Correlation between concurrent values is not required.

It is possible to obtain as good an agreement by means of either (1) related hydrologic data readily obtained and used directly, such as monthly rainfall or a nearby record of peak discharges, (2) unrelated hydrologic data, or (3) a random sample of numbers drawn from a distribution similar to that of the data to be duplicated. Examples of each of these types will be illustrated.

a. Proc. Paper 1451, December, 1957, by J. L. H. Paulhus and J. F. Miller.
1. Hydr. Engr., U. S. Geological Survey, Washington 25, D. C.
2. Hydr. Engr., U. S. Geological Survey, Washington 25, D. C.

Fig. 1 is a plot of annual floods on Codorus Creek against annual maximum monthly precipitation at York, Pa. The relation is expressed by a straight line fitted by eye. From this relation the synthesized annual flood corresponding to each annual maximum monthly precipitation value was obtained. These values are plotted against the observed values in Fig. 2 for comparison. That figure also compares the authors' synthesized floods with the observed floods. Although a simple correlation of monthly rainfall gives nearly as reliable estimates of annual floods as the authors' laborious methods, the writers do not advocate estimating flood peaks from monthly rainfall. The purpose of these comparisons is to show that either set of synthesized floods will define about the same frequency curve as is defined by the observed floods. Table 1 shows arrayed values of flood peaks for Codorus Creek as observed, as synthesized by the authors, and as synthesized using monthly precipitation.

Table 2 shows annual flood peaks for Allegheny River at Red House, N. Y. which bear no relation to simultaneous annual flood peaks on Codorus Creek. In this example the Allegheny River peaks are arrayed before correlating, in order to increase the accuracy of synthesizing peaks. This process is similar to that of the authors in that the ranking procedure is used once, although this time it is used prior to establishing the relation curve. Fig. 3 shows the relation between ranked values. From this relation the synthesized floods for Codorus Creek (listed in Table 1) are obtained.

Similar agreement between ranked peaks, synthesized and observed, has been obtained using a set of random numbers. In Fig. 4, the observed annual floods for Clark Fork below Missoula, Mont. for the period 1930-48 are plotted on logarithmic probability paper and indicate a log-normal distribution. To duplicate this distribution, a set of random normal numbers was selected from a table and plotted in the order drawn against the logarithms of the observed floods in chronological order. The plot is shown in Fig. 5. The straight-line curve shown was drawn through two points defined respectively by the lowest and highest values in each set of data (this equalizes the range of the observed and synthesized floods). Although there is no correlation between the two sets, this curve is used to obtain values of synthesized floods. Agreement between the observed and the synthesized floods so obtained is shown on Fig. 4.

Although the frequency curve for the period of record can be reproduced readily from any of several types of data it does not follow that synthesized floods for years outside the period of record will reproduce the frequency curve that would have been obtained by using actual annual floods. To reproduce that curve without an assumption of similarity of distribution requires accurate estimates of individual flood events. The authors' method might have been evaluated by applying it to the latter part of a long flood record and testing the extended results against the known earlier part of the period. No verification of extrapolated results can be made for their example.

The authors admit that the second and third columns of their Table 2 "show poor agreement on an annual basis between observed and synthesized floods." They have in this paper abandoned the idea of using hydrologic data casually related, such as rainfall records within the same basin, in favor of using data only indirectly related (i.e., rainfall records outside the basin). If this approach is justified, simpler methods, using either rainfall or flood discharge at other stations, might give equally good or better results.

This discussion has ignored the question as to whether the desired goal should properly be the extension of record at a single location. Such a record

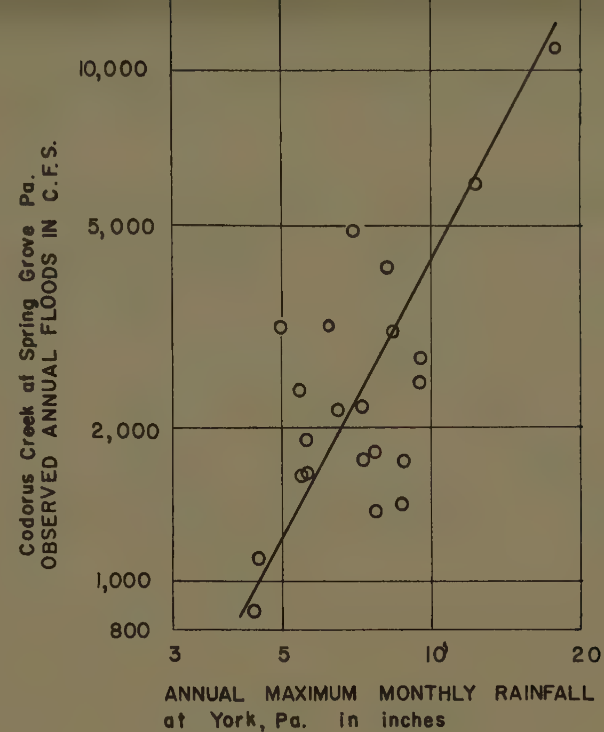

Figure 1- Relation between annual floods and annual maximum monthly rainfall, 1933-54

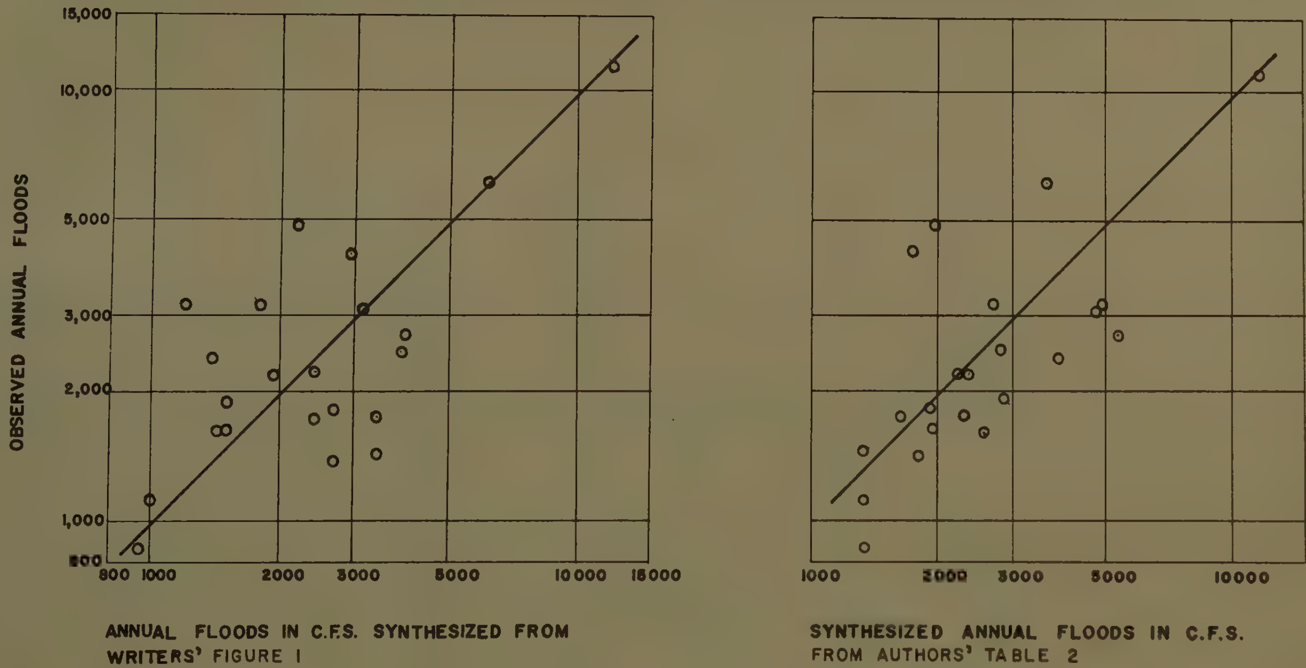

Figure 2- Comparison of observed and synthesized annual floods, Codorus Creek at Spring Grove, Pa., 1933-54

Table 1- Arrayed annual floods
for Codorus Creek at Spring Grove, Pa.

Order number	Observed (in cfs)	Synthesized (in cfs) By Authors	From monthly precipitation	From Allegheny R. peaks
1	11,200	11,600	12,400	11,200
2	6,070	5,370	6,200	5,400
3	4,880	4,900	3,950	4,800
4	4,180	4,800	3,900	4,050
5	3,200	3,910	3,400	3,400
6	3,180	3,670	3,400	3,100
7	3,070	2,890	3,150	3,050
8	2,710	2,810	2,950	3,000
9	2,490	2,730	2,700	2,900
10	2,380	2,590	2,700	2,250
11	2,210	2,360	2,450	2,150
12	2,180	2,340	2,430	2,050
13	1,910	2,240	2,200	2,000
14	1,810	1,970	1,950	1,700
15	1,750	1,940	1,800	1,700
16	1,730	1,930	1,500	1,600
17	1,620	1,780	1,500	1,600
18	1,610	1,750	1,420	1,550
19	1,430	1,640	1,400	1,550
20	1,380	1,340	1,200	1,350
21	1,110	1,330	990	1,250
22	870	1,320	930	870

Table 2- Annual peak discharges for
Allegheny River at Red House, N. Y.

Water Year	As observed (cfs)	Ranked (cfs)
1933	15,500	45,300
1934	18,800	38,500
1935	19,300	37,000
1936	30,700	34,900
1937	24,400	32,200
1938	20,400	30,700
1939	25,200	30,400
1940	30,400	30,100
1941	20,200	29,900
1942	45,300	25,200
1943	29,900	24,400
1944	18,700	23,800
1945	23,800	23,300
1946	34,900	20,400
1947	37,000	20,200
1948	38,500	19,300
1949	10,300	19,200
1950	30,100	18,800
1951	32,200	18,700
1952	23,300	16,600
1953	19,200	15,500
1954	16,600	10,300

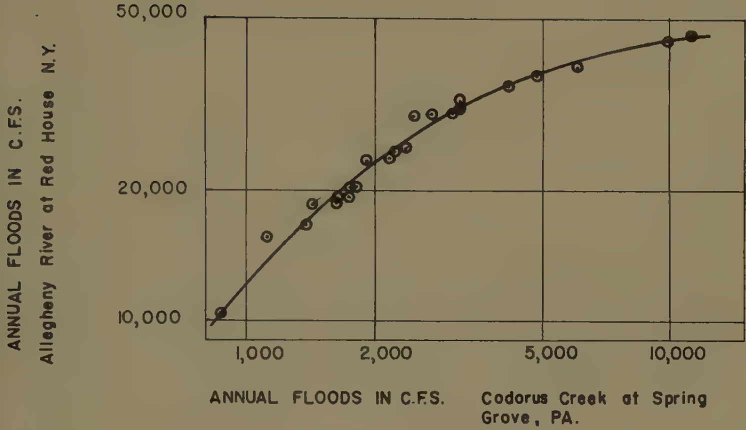

Figure 3- Relation between equal-ranked peak discharges

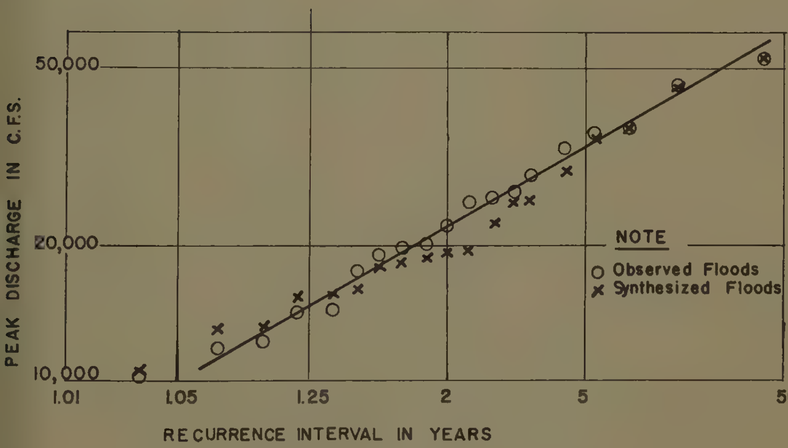

Figure 4- Flood-frequency curve for Clark Fork
Missoula, Mont., 1930-48

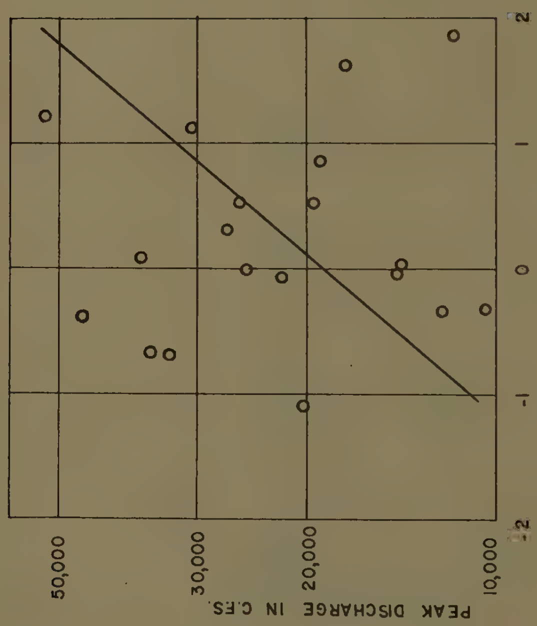

Figure 5- Plot of annual floods, Clark Fork below Missoula, Mont., 1930-48, against 19 random numbers

known to have a wide range of sampling error, even when obtained by direct servation. This type of error may be minimized by using regional methods nich combine many records and provide frequency information not only at ged sites, but at ungaged sites as well.

DISCHARGE CHARACTERISTICS OF RECTANGULAR THIN-PLATE WEIRS[a]

Discussions by Steponas Kolupaila, Ralph W. Powell, John W. Paull and Iwao Oki

STEPONAS KOLUPAILA.[1]—Investigation of weirs is one of the most popular experiments in hydraulic laboratories. In the treasury of 122 volumes of ASCE Transactions there are not less than 100 valuable papers and important discussions on water flow over the weirs. The best contribution was the paper by E. W. Schoder and K. B. Turner, "Precise Weir Measurements", 1929. Since that time new experiments were made, more empirical formulas published, contradictions discussed and sources of errors investigated. Triangular, circular, parabolic and hyperbolic notches proved to be more accurate and reliable for small water rates. The rectangular weir remains unconquerable for larger discharges. The majority of known formulas assure ± 2% accuracy under condition of adequate arrangement. A search for higher precision was the purpose of new investigation, conducted by the authors at the Georgia Institute of Technology. A series of 249 new tests was performed and some previous results were reviewed and collated. The sponsor of the project and attentive readers expected a new revelation in the form of a modern dimensionless formula correct to ± 5%. Regretfully, the conclusions of the authors are rather discouraging: a truly reproducible, standard measuring weir and a precise, universal discharge formula are impractical.

The writer cannot agree with this pessimistic view and with some other statements by the authors. For example, the opinion that the flow pattern for rectangular weirs is not subject to complete mathematical analysis, is not entirely true. The flow over the weir must be studied as

$$Q = \int_A v \cos \phi \, dA,$$

where ϕ is the variable angle of stream lines with the normal to the cross section A. A certain law of stream lines is to be established or approximation assumed, before the integration can be done. Finally, a small correction factor may be introduced to account for secondary influences, as wall roughness, edge character, surface tension and viscosity. G. Kirchhoff applied this method (1883) to a two-dimensional slot in the bottom of a container and derived a well known theoretical value of

$$c_d = \frac{\pi}{\pi + 2} = 0.611.$$

a. Proc. Paper 1453, December, 1957, by Carl E. Kindsvater and Rolland W. Carter.

1. Prof. of Civ. Eng., Univ. of Notre Dame, Notre Dame, Ind.

Work by Kirchhoff was continued by F. Kötter in 1887. L. A. Ott (1932) tried to use an approximate pattern, assuming concentric circles as equipotential lines, for orifices, sluices and weirs. He derived for a weir, as a first approximation, the value

$$c_d = \frac{2}{\pi} = 0.637.$$

This field is free for further development.

Velocity distribution over the crest of the weir was investigated, as the most important factor, by Ch. Keutner in his thesis (1929) and in several papers. He used projections v cos ϕ and derived a set of formulas taking in account velocity laws.

The authors term it a misinterpretation by those who associate the coefficient k, with the kinetic energy (Coriolis) coefficient α. Their opinion is based on usually assumed values of α of about 1, while k, in the Bazin derivation is taken as 2. Actually the velocity distribution approaching the weir is very irregular and α can be much greater than 2. H. Lauffer (1935) computed values for several velocity diagrams in the Shoder and Turner work and obtained 1.64, 1.85 and 1.99. Values as high as 3.87 and even 7.4 before a turbine are mentioned in the literature (Shchapov, 1957). We know little about the Coriolis coefficient; for this reason the writer presented the methods of determination of this important factor (1956).

Readers, who use the metric system, would be more satisfied, if data in this paper would be dimensionless, easy to compare, e.g., if values of c_e would be divided by $\sqrt{2g}$. This would simplify the comparison with many European tests and discussions, as by R. Hailer (1929), G. Proskura (1931), W. Dietrich (1932), O. Dillmann (1933), A. Testa (1934), F. Engel (1935), T. Fendt (1936), B. Gentilini (1936), H. Gerber (1937), F. Paderi (1939), A. Vitols (1938, 1942), Ja. T. Nen'ko (?), G. L. Dolidze (1952) and others.

The most comprehensive paper on the weir with lateral contractions was presented by E. Zschiedrich in 1939, as a thesis at the Technical University Dresden; there is also a Russian study by A. I. Bredis.

Results of high quality were gained by O. Kirschmer and B. Esterer in the great Walchensee hydraulic laboratory in 1929. The writer compared these with Georgia Tech tests and found some difference. Surprisingly, the conclusion of the Walchensee tests was expressed as being unfavorable by the authors. It appears as though the rectangular weir is in danger as a hydrometric device.

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RALPH W. POWELL,[1] M. ASCE.—This is one of the oldest problems of hydraulics, and one on which a vast amount of effort has been expended. The number of formulas which have been proposed may run into the hundreds, and most of them are almost unusably complex. The formulas here proposed are refreshingly simple, and seem to be satisfactory for water at ordinary temperatures. A complete treatment, including the effect of viscosity and surface tension, such as Lenz(1) has given for the V-notch, must await experiments on the flow of other liquids.

Putting $h_e = h + 0.003$ in the authors' Eq. (11) and expanding by the binomial theorem and dropping terms which are insignificant gives

$$Q = C_e\ (1 + 0.0045/h)\ b_e h^{1.5}$$

1. Visiting Prof., Applied Mechanics, Univ. of Kansas, Lawrence, Kans.

Substituting the authors' Eq. (29) in this we have

$$Q = (3.22 + 0.44\ h/P + 0.0145/h + 0.0020/P)\ b_e h^{1.5}$$

which is very similar to Rehbock's 1912 formula. In fact, if K_h is taken as 0.0037 instead of 0.003, P as 0.5 ft., and b_e as b(1-.003), we have

$$Q = (3.23) + 0.44\ h/P + 0.0178/h)\ b\ h^{1.5}$$

which is almost identical with the authors' Eq. (19). But the authors' form of stating the equation is obviously simpler.

Treating the full-width weir as the limiting case of the rectangular notch is new to the writer, but seems justified in view of the data given by the authors. Their use of a crest only 0.30 ft. above the floor for some of their tests is unusual. Since they wished to find the proper coefficient of the h/P term in the formula, it was wise to make the term large by making P small. But as we can never be sure that we have found the best value of this coefficient, the writer believes that P should ordinarily be made at least 2 ft. The authors find that their experiments give the value of the coefficient of h/P to be 0.40, and that the USBR tests gave only 0.30. But as the Bazin tests and Rehbock's 1929 formula give 0.44 and Schoder and Turner's careful and very extensive tests at least that much, the writer is using 0.44 throughout this discussion.

Although the height of the weir should not be too small, Cline(2) pointed out some years ago that it is unnecessary to make weir boxes as deep as had been thought necessary. In fact, there seems a definite disadvantage besides expense in making the weir box much deeper than the approach channel, or if it is supplied by a pipe, in having the weir more than 2 or 3 ft. high. The reason is that it is difficult to distribute the velocity of approach in a stable manner. Regions of high velocity tend to form and move about in a deep weir box. When these are along the bottom, the water approaching the weir has an upward velocity which raises the lower nappe and decreases the discharge. When the higher velocity is along the surface, the contraction is decreased and the discharge increased, even though the head on the weir remains exactly the same.

The authors have emphasized the importance of the velocity distribution in the weir box, but they have given no quantitative information as to the error resulting from having a distribution other than "normal". During the past months the writer has been faced with this problem and wishes to suggest a solution. The particular problem was to calculate flows over a full-width weir 8 ft. long, with P = 5.5 ft., at the end of a weir box 45 ft. long. The water entered from a semicircular flume, 6 ft. in diameter, with its bottom about 3 ft. above the bottom of the weir box. When the water was turned on a current flowed along the floor of the weir box which tended to persist even after the box was filled to a depth of 6.5 ft. and water was flowing over the weir with a head of 1 ft. This in spite of a baffle that had been placed near the upstream end.

The writer's first thought was that all that was necessary was to make a velocity traverse at the section where the head was being measured (14 ft. upstream from the weir), and use Schoder and Turner's formula,

$$Q = 3.33\ B \left[\left(h + \frac{V_a^2}{2g}\right)^{1.5} + \frac{h}{3.33} \times \frac{V_b^2}{2g} \right] \qquad \text{(D)}$$

where V_a is the mean velocity of approach in the part of the cross-section above the level of the weir crest, and V_b is the mean velocity of approach in the part below the level of the weir crest. But it was noted that this formula always made Q more than 3.33 $Bh^{1.5}$ although the data on which it was based showed four cases where the observed Q was less than 3.33 $Bh^{1.5}$ and many others where it was less than shown by (D). These tended to be runs where V_b was more than V_a, and since this was true in the case under consideration, it was thought best to make a new study of the original data. The first result was not very satisfactory,[3] and since reading the authors' paper, a second study has been made as follows.

It was assumed that $C_e = 3.22 + 0.44 \frac{h}{P} + \phi(r)$ where $r = V_a/V_b$ as defined above and $\phi(r)$ is an unknown function to be determined. Then 72 of the Schoder and Turner runs for which the velocity distribution had been measured were selected. These included all of the 36 runs in which the velocity distribution had been changed by "fences" and an equal number of runs without fences. They included runs from Series D, E, F, G, H, I, J, K, L, M, N, and O. Assuming that $b_e = b - 0.003$ and $h_e = h + 0.003$, C_e was computed for each of these runs, and from it $\phi(r)$ was computed as $C_e - 3.22 - 0.44\ h/P$. These values were plotted against r as in Fig. 1. Since this small residual includes all the experimental errors, there was of course a good deal of "scatter", but a line drawn through the points by eye had the equation,

$$\phi(r) = 0.15r - 0.14 \tag{E}$$

This leads to the formula

$$C_e = 3.08 + 0.44h/P + 0.15r \tag{F}$$

Values of C_e computed by this formula were compared with the C_e's found from the observed data for each of the 72 runs. The average discrepancy was 1.41% as compared with 1.09% for the same 72 runs figured by Schoder and Turner's Eq. (D). However, the algebraic sum of the 72 discrepancies was 17.56% by formula (D) and only 7.37% by formula (F). If we confine our attention to the 14 runs in which V_b was more than V_a, which was the case in which we were especially interested, Eq. (F) gives a mean discrepancy of 1.70% and a total algebraic discrepancy of 10.47%, which Eq. (D) gave a mean discrepancy of 2.15% and a total algebraic discrepancy of 22.8%. If we include only the eight of these runs in which P was 2.0 ft. or more, Eq. (F) gives a mean discrepancy of 0.47% and a total algebraic discrepancy of 0.33%, while Eq. (D) gives a mean discrepancy of 1.22% and a total algebraic discrepancy of 5.51%. It is felt therefore that Eq. (F) is a more satisfactory empirical formula than Eq. (D), especially since it is simpler to compute.

In these runs, r varied from about 0.4 to about 4.0. which made C_e vary from 3.14 + 0.44h/P to 3.68 + 0.44h/P or 17.2% resulting from changes in the velocity distribution alone. In series C the actual individual runs vary even more than this, for on page 1065 of reference (7), Schoder and Turner say:

> "For the same head, an extreme difference of 26% in discharge was produced by variations in the velocity distribution . . . No doubt careful inspection of the appearance of the flow in the channel of approach would enable cases of extremely high surface or bottom velocities to be noticed by an experienced engineer. It seems certain, however, that no visual inspection could distinguish between discharges that differ as much as 4 or 5 per cent at the same head due solely to different distributions of velocities in the channel of approach."

Although they referred to Eq. (D) as "perhaps the best tentative formula [which they] found" and admitted that there were still some fairly large discrepancies, some have regarded it as a very accurate formula, and not used simply because of the difficulty of having to measure V_a and V_b. In this connection it may be mentioned that Francis M. Dawson, M. ASCE, who had a large part in the preparation of Schoder and Turner's paper, has pointed out to the writer that it is a misconception that V_a and V_b must be measured for each run, for if the approach arrangement remains unchanged, a few gagings for various heads will determine curves giving V_a and V_b as functions of h which will be accurate enough to use in the formula. The same applies even more to Eq. (F), as here only one curve, V_a/V_b as a function of h, is required. In the writer's problem referred to above, after additional baffles had been installed near the upstream end of the weir box to improve the velocity distribution, it was found that V_a/V_b could be taken as 0.435h + .870, which with P = 5.5 ft., reduces Eq. (F) to $C_e = 3.210 + 0.145h$ or

$$Q = (3.210 + 0.145h)\,(b - 0.003)\,(h + 0.003)^{1.5}$$

It must be confessed, however, that we have oversimplified the problem. For a high weir the same value of V_b may represent quite different distributions, some with maximum near the bottom, and some with the maximum near the top. This is probably the principal reason for the large scatter of the points in Fig. 1. It is probable that the ratio of the velocity at 0.1 depth to that at 0.9 depth would be a much better criterion on which to base the "distribution of velocity of approach" correction. This would require a whole new series of experiments. In the meantime errors of the order of one per cent or more can be expected from weir measurements. Rehbock's statement that the mean discrepancy should not exceed 0.2 or 0.3% seems too optimistic except for exactly the same entrance conditions to exactly the same weir box and weir as he used.

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JOHN W. PAULL.[1]—The paper is a good example of modern scholarship. Dimensional analysis has been used to obtain a fundamental solution to a fluid flow problem for which only empirical formulae were available before.

1. Lecturer—Hydraulic Eng., Dept. of Civil and Aeronautical Eng., Royal Melbourne Technical College, Melbourne, Australia.

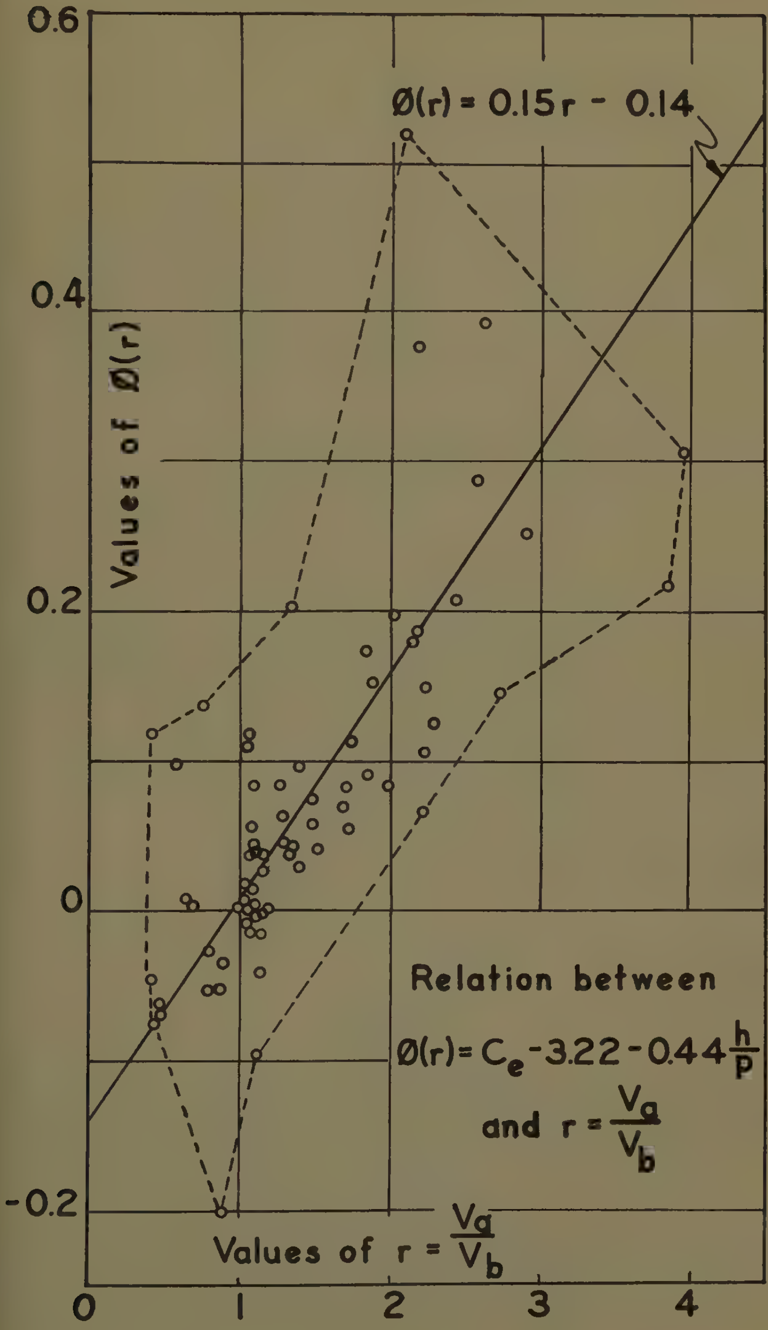
0.6
0.4
0.2
0
-0.2
0
1
2
3
4
Values of Ø(r)
Ø(r) = 0.15r - 0.14
Relation between
Ø(r) = Ce - 3.22 - 0.44 h/p
and r = Va/Vb
Values of r = Va/Vb

Fig. 1

The discharge equation selected by the authors,

$$Q = C_e b_e h_e^{3/2}$$

where C_e = a coefficient depending on $\frac{h}{P}$ and $\frac{b}{B}$,

b_e = effective width of weir = $b + k_b$,

h_e = effective head on weir = $h + k_h$,

is very simple. As the C_e term takes into account what has been termed 'the velocity of approach', and k_b and k_h take into account the effects of viscosity and surface tension, a very simple way has been devised for eliminating complexities.

After clarifying and simplifying the picture in regard to flow over a rectangular, thin plate weir the authors conclude with a sober note: "a truly reproducible, standard measuring weir and a precise, universal discharge formula are impractical."

A valuable addition to the paper would have been a more definite indication of the accuracies to be expected under a variety of conditions.

IWAO OKI.[1]—The writer has read this paper with much interest. The proposed comprehensive solution for the discharge characteristics of rectangular, thin-plate weir is very adequate and future students in this field will be greatly benefited by the authors' achievement.

The writer contributed, first in 1929, a paper(1) on "An Empirical Formula for the Discharge over Rectangular Weirs", the object of which was to establish an empirical formula which had a wider range of applicability and could be conveniently used in practice.

From several formulae known at that time the writer selected two typical ones. The one was the Francis formula,

$$Q = 3.33\,(b - 0.2h)\left\{ (h + h')^{3/2} - h'^{3/2} \right\},$$

in which h' is the head due to the velocity of approach.

The other was the Barnes formula,

$$Q = 3.324\, h^{1.49}\, b^{1.11}\,(b + 2h)^{-0.11},$$

in which h is observed head + $(1/70)\,u^2$, u being the mean velocity in the approach channel.

When these formulae were rewritten in the form,

$$Q = C\,\frac{2}{3}\sqrt{2g}\;\; bh^{3/2},$$

the ratio h/b seemed to be a predominant parameter in the coefficient C. By comparison of the results of many experiments, including some on a model in Japan, the writer suggested the following expression for the coefficient C,

$$C = 0.6224\,(1 + \frac{0.004}{h}\left\{1 - \frac{\sqrt{m}}{10}\,(1 - \frac{m}{3p})\right\}\left\{1 + \frac{1}{2}\left(\frac{bh}{B(p + h)}\right)^2\right\}$$

where m stands for the ratio h/b and P denotes the height in feet of the weir crest above the bottom and B the width of the channel. Adopting 0.623 instead

1. Emeritus Prof. of Waseda University, Tokyo; Member of Japan Academy.

of the constant 0.6224, this formula has been widely in use in Japan.

In 1935, the writer prepared a paper(2) on a method of studying more profoundly the coefficient C for discharge. The values of the coefficient are affected by so many factors that they have not yet been exactly determined, notwithstanding the elaborate experiments by many authors. The writer thought the first thing to be done was to determine the coefficients of discharge for weirs with absolutely complete contractions.

From the most reliable results of experiments on rectangular weirs with two side-contractions, including those in the Hydraulic Laboratory of the Waseda University, the writer picked out those cases where it was considered the flows were entirely free from bottom—and side wall—effects. The coefficients of discharge thus selected were arranged in groups for the same values of the ratio $h/b = m$, against $1/b$, the reciprocal of the weir length. The curves drawn, as shown in Fig. 1, through the points for each group were extended until they intersect the axis of ordinate where b is infinite. The values of the coefficients at the intersections were denoted by C_∞, which represented those for absolutely complete contractions, since in those cases the influences by all the factors, except the ratio $h/b = m$, upon the values of discharge coefficients becomes negligibly small. The coefficients C_∞ were plotted, as in Fig. 2, against the ratio $h/b = m$. To make fit with these values of C_∞, the writer deduced, in 1954, a new formula,(3)

$$C_\infty = 0.612 \left(1 - \frac{\sqrt{m}}{10 + 1.8m^2}\right)$$

for rectangular weirs with absolutely complete contractions. For m = 1 the coefficient takes a value $C_\infty = 0.560$, and when m = 1.36 the coefficient attains a minimum value $C_\infty = 0.558$. 0.612 is the value of the coefficient C_∞ when m = 0, also, $= \infty$. If the potential flow theory in two dimensions might hold in these cases, the constant 0.612 would be replaced by 0.611.

The writer hopes that the propositions stated above will contribute something to the completion, in the future, of an equation which covers the discharge over rectangular, thin-plate weirs of all sizes and proportions.

In Japan many investigators are endeavoring to establish an adequate formula for the discharge over rectangular weirs and there has been compiled much data in this field of hydraulics, some of which, are highly valuable.

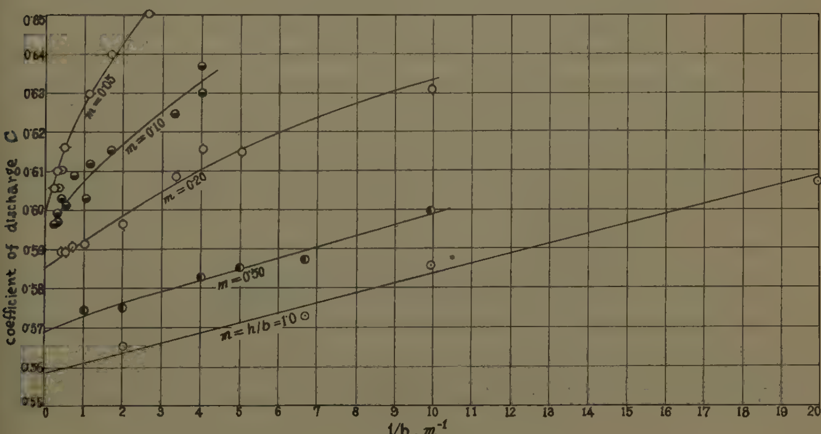

Fig. 1

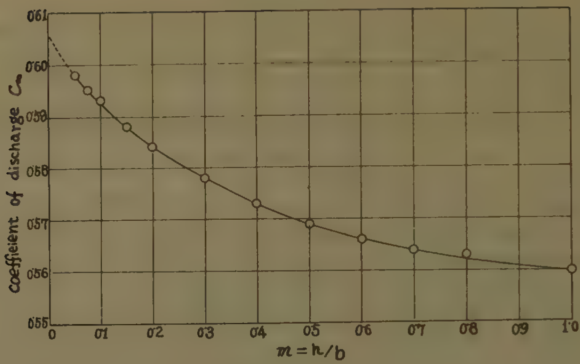

Fig. 2

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2. Oki, Iwao, A Method of Studying the Coefficients of Discharge for Rectangular Weirs with Two Side-Contractions, Transaction of the Society of Mechanical Engineers, Japan, Vol. 1, No. 4, Oct. 1935.
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AIR BINDING IN LARGE PIPE LINES FLOWING UNDER VACUUM[a]

Discussion by R. E. Templeton and T. E. Stelson

R. E. TEMPLETON,[1] J. M. ASCE and T. E. STELSON,[2] A. M. ASCE.—The escription of two situations where air binding caused excess head loss in arge pipelines flowing under partial vacuum is extremely interesting. Such onditions of hydraulic inefficiency frequently go undetected, and hence, unorrected. The excess head loss may often be attributed to higher roughness oefficients than had been assumed in design computations. Such confusion asts doubt on the precision of hydraulic computations, whereas the real deiciency is in an understanding of the flow conditions.

The statement is made that ". . . for a given water velocity and pipe shape ir binding difficulties increase with the pipe size." Laboratory experiments nd theory support this conclusion. When the velocity distribution and mean elocity are the same, the shearing stresses tending to move air bubbles along he top of the pipe will vary inversely as the pipe diameter. Hence, air bindng will occur in a five foot diameter pipe, having a mean velocity of 10 feet er second, in approximately the same way as it will occur in a one foot dimeter pipe having a mean velocity of two feet per second.

Mr. Richards has stated that "It is apparent from the comments above that ir evacuation equipment must be used unless a change in grade is made by a ertical drop." The writers disagree with this statement. Air binding occurs n a vertical drop in much the same way as it occurs in a gradual downgrade. 'or a given flow of air and water, a vertical drop will cause less head loss an will a gradual downgrade. Hence, the air binding still exists but the onsequences of it are not as serious.

In tests at Carnegie Institute of Technology,[3] the characteristics of flow of ir-water mixtures past transitions from horizontal to downgrade pipes were tudied. The tests were conducted on pipes having an inside diameter of 5.59 nches. The transition from horizontal to downgrade was formed by the interection of the two cylindrical pipe surfaces with axes in the same vertical lane. In one case, the pipe grade was changed from horizontal to vertical. n the other case it was changed from horizontal to a downward inclination of wenty-nine degrees from the horizontal.

Visual observations through the transparent pipes and measurement of ressure showed that y, the vertical height of air binding, which is

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. Proc. Paper 1454, December, 1957, by R. T. Richards.
. Design Engr., Piping Div., M. W. Kellogg Company, New York, N. Y.
. Associate Prof. and Acting Head, Carnegie Inst. of Technology, Pittsburgh, Pa.
. Templeton, R. E., "Air Entrainment in Pipes and Jets," M.S. Thesis, Carnegie Institute of Technology, June 1954.

approximately equal to the head loss, to be the following functions of Q_a to Q_w, the volumetric ratio of air discharge to water discharge:

Horizontal to Vertical (90°) Transition

$$y = 14.1 \frac{Q_a}{Q_w}$$

Horizontal to 29° Downgrade Transition

$$y = 41.7 \frac{Q_a}{Q_w}$$

Thus, for the same ratio of air to water discharge, air binding in the vertical drop causes approximately only one-third as much head loss as does air binding in the twenty-nine degree change in grade. Vertical drops would thus reduce the hydraulic inefficiency but would not remove the air binding problem. The removal of air at vertical drops may be required and the installation of a series of vertical drops will not solve an air binding problem. This further supports the author's conclusion that an arrangement of vertical drops is not a satisfactory solution to air binding problems because of elbow loss and the development of large thrust forces.

Air binding can be as serious a problem in conduits flowing under positive pressure as in conduits flowing under negative pressure. For example, in oil refineries, vapor binding can occur in return lines from reboilers that supply heat to different types of towers and in lines downstream from control valves where an equilibrium liquid has flashed into a vapor liquid mixture. The only difference is that the negative pressures tend to increase the air discharge if air enters the pipes through leaks and/or comes out of solution. Furthermore, the removal of air under vacuum is far more difficult.

Considerably more laboratory study and field measurements are necessary before the flow of liquid gas mixtures is fully understood.

FLOW THROUGH CIRCULAR WEIRS[a]

Discussions by Madhav Manohar, Fred W. Blaisdell, W. T. Moody, Jean Rigard and M. B. McPherson

MADHAV MANOHAR,[1] J. M. ASCE.—The author is to be commended for resenting a rigid accurate formula for flow through circular weirs.

In 1952, at the University of Minnesota, the writer had occasion to resort the same approach in the analysis of part-full flow through a sharp edged ılet of a pipe culvert. The study(1) was, then, mainly concerned with the hy-raulics of part-full, and transitional flow through pipe culverts equipped with ifferent types of inlets and laid on steep slopes. When a culvert is laid on teep slopes, the inlet becomes the control section for part-full flow so that hanges in slope do not have any effect on headwater elevation and dis-harge.(1) In the case of a well-rounded inlet, critical depth occurs at the ntrance and for a sharp-edged round inlet, flow may be regarded as if it is ırough a circular weir. Using the same procedure and the resulting equations ıvolving elliptic functions as derived by the author, the writer was able to btain a stage-discharge relationship for the flow through a sharp edged inlet s shown in Fig. 2 with an average coefficient of discharge, C = 0.519 (Fig. 1 nd Table 1). The experiments were conducted accurately in a 35' long, 4" iameter tilting pipe. Irrespective of the steep slopes on which the culvert as laid, the variation in the coefficient of discharge from the mean value of .519 was very slight (Fig. 1). For convenience of calculations, the elliptic ınctions K and E were plotted as functions of H/D and ψ_o (Fig. 3).

The writer believes that the lower value of C of 0.519 obtained by the riter as against 0.59 of the author may be from the effects of the culvert in-ert at the inlet on the headwater elevation preventing free fall of the flow om the inlet.

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Proc. Paper 1455, December, 1957, by J. C. Stevens.

Prof. and Head of Civ. Eng. Dept., Birla Inst. of Technology, Ranchi, India.

TABLE 1: Coefficient of Discharge:

Sharp Edged Inlet Acting as Circular Weir.

Diameter of Inlet = 0.333 ft.

H ft	H/D	Q_t cfs	Q cfs	C	H ft	H/D	Q_t cfs	Q cfs	C
Slope = 4.10 %					Slope = 5.30 %				
.183	.549	.103	.049	.476	.114	.342	.042	.022	.524
.223	.669	.146	.071	.486	.164	.492	.086	.045	.523
.242	.726	.168	.086	.511	.202	.606	.123	.063	.512
.267	.801	.199	.100	.502	.223	.669	.146	.075	.513
.281	.843	.215	.109	.506	.245	.735	.172	.089	.517
.298	.894	.233	.119	.510	.271	.813	.202	.104	.515
.310	.930	.246	.127	.516	.287	.861	.222	.114	.515
.322	.966	.262	.137	.523	.301	.903	.235	.128	.544
					.317	.951	.240	.134	.559
					.333	.999	.276	.145	.525
			C_{ave} =	.504				C_{ave} =	.525
Slope = 5.90 %					Slope = 6.55 %				
.111	.333	.039	.022	.564	.118	.354	.045	.022	.490
.201	.603	.122	.063	.516	.174	.522	.095	.048	.505
.240	.721	.166	.084	.506	.215	.645	.136	.070	.515
.265	.795	.196	.099	.505	.238	.714	.164	.086	.524
.282	.846	.215	.109	.507	.258	.774	.189	.097	.513
.293	.879	.229	.117	.511	.272	.816	.204	.106	.520
.307	.921	.243	.128	.527	.288	.864	.222	.114	.514
.319	.957	.257	.134	.521	.303	.909	.238	.126	.529
.333	.999	.275	.147	.531	.318	.954	.256	.136	.530
					.330	.990	.276	.144	.522
			C_{ave} =	.521				C_{ave} =	.516

H ft	H/D	Q_t cfs	Q cfs	C	H ft	H/D	Q_t cfs	Q cfs	C
Slope = 7.20 %					Slope = 7.90 %				
.100	.300	.033	.022	.666	.113	.339	.041	.022	.536
.150	.450	.071	.035	.493	.149	.447	.071	.036	.508
.195	.585	.115	.059	.513	.179	.537	.120	.052	.433
.227	.681	.150	.077	.513	.229	.687	.153	.077	.502
.255	.765	.185	.096	.518	.245	.735	.172	.089	.516
.277	.831	.210	.110	.524	.263	.789	.194	.101	.521
.297	.891	.232	.122	.526	.281	.843	.215	.112	.521
.310	.930	.247	.129	.522	.298	.893	.231	.122	.528
.333	.999	.276	.145	.525	.311	.933	.248	.136	.548
			C_{ave} =	.533	.333	.999	.276	.145	.525
								C_{ave} =	.514

Average of all values of C = 0.519.

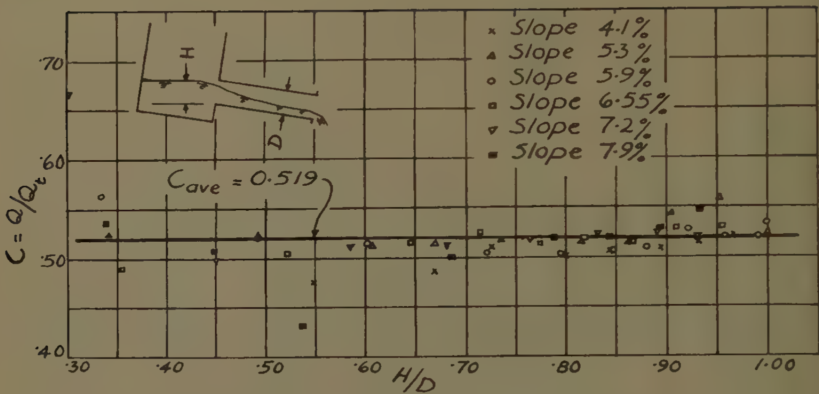

FIG. 1: COEFFICIENT OF DISCHARGE, C, AS FUNCTION OF H/D

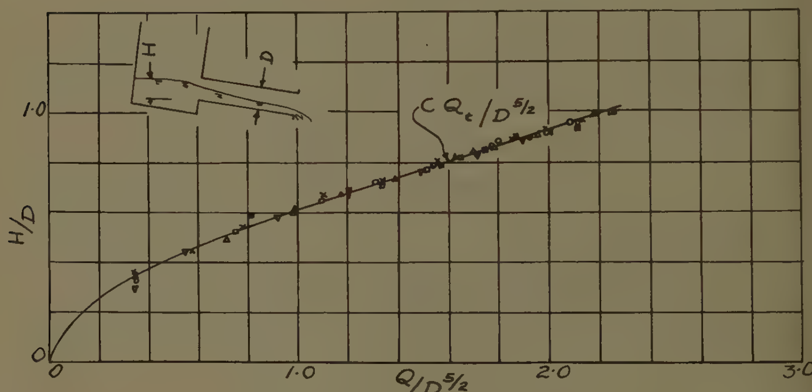

FIG. 2: STAGE-DISCHARGE RELATIONSHIP

Culvert with sharp-edged inlet: Circular Weir theory

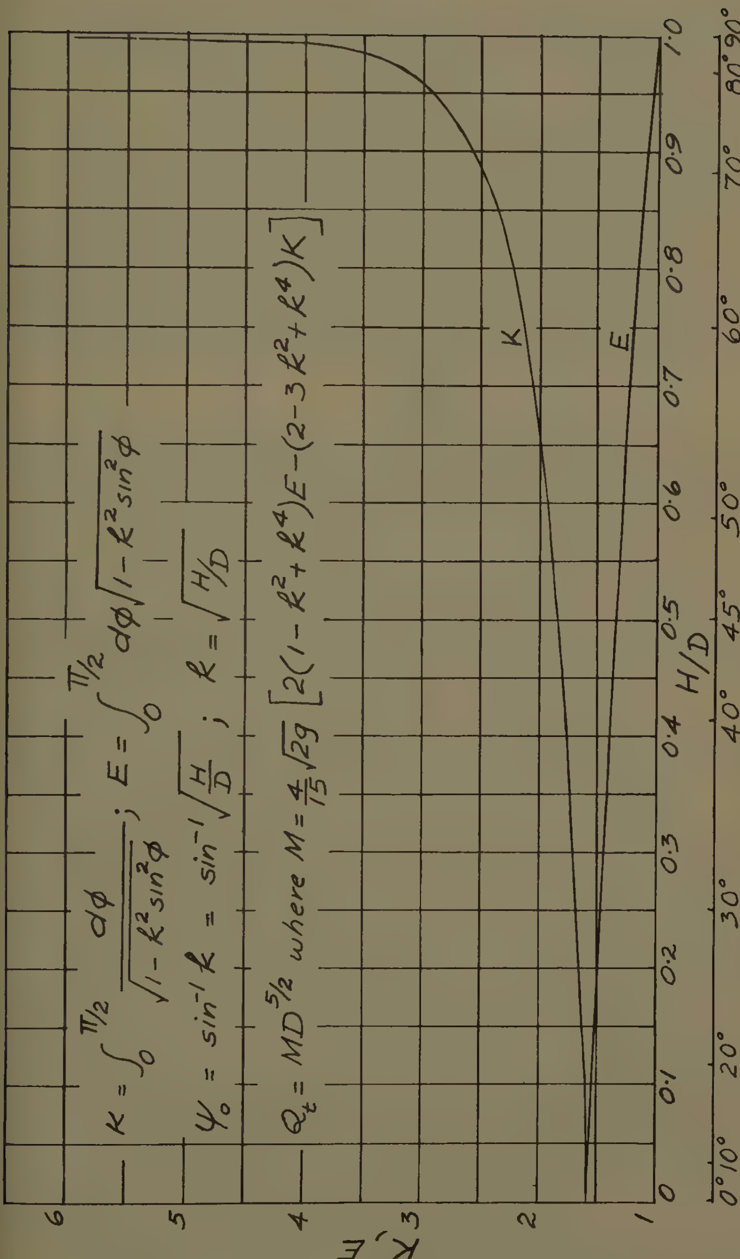

FIG. 3: ELLIPTIC FUNCTIONS, K AND E, AS FUNCTIONS OF H/D

FRED W. BLAISDELL,[1] M. ASCE.—Mr. Stevens has summarized his source data in Appendix A. This used to be a common practice that has been largely abandoned in recent years. It is hoped that Mr. Stevens' example will be followed by others. Besides making original data readily available to all who have use for it, it facilitates discussion and bespeaks the author's confidence in his presentation and his willingness to let others independently evaluate his conclusions.

It is to be regretted that the author has not more fully described the approach to each weir, for it is well known that a weir head-discharge relationship is influenced by the approach conditions. Such information would be of considerable value to others in evaluating the results.

Mr. Stevens is to be commended for his elegant solution of the theoretical equation for flow through circular weirs and his preparation of the table of Δ without which his Eq. (3) would probably find little use in practice.

To satisfy his own curiosity, the writer has used a different approach and has evaluated it using the data assembled by Mr. Stevens in his Appendix A. The analysis starts with the basic discharge formula

$$Q = ca\sqrt{2gH} \tag{A}$$

where, in addition to the symbols defined by Mr. Stevens, a is the area of flow corresponding to the head H. Eq. (A) can be put into dimensionless terms by dividing both sides by $\sqrt{2g}\ D^{5/2}$ which gives

$$\frac{Q}{\sqrt{2g}\ D^{5/2}} = c\,\frac{a}{D^2}\sqrt{\frac{H}{D}}$$

Substituting for D^2 its equivalent, $4A/\pi$ where A is the area of the orifice,

$$\frac{Q}{\sqrt{2g}\ D^{5/2}} = c\,\frac{\pi}{4}\,\frac{a}{A}\sqrt{\frac{H}{D}} \tag{B}$$

Tables giving values of a/A as a function of H/D are available in a number of publications.(1)

Values of c, computed according to Eq. (B) from the data presented in Appendix A are plotted in Fig. (A). The arrangement of the plot is similar to that of Mr. Stevens' Fig. 2 to facilitate comparison. The principal difference that can be readily noted is that the coefficients in Fig. A increase at high values of H/D whereas this tendency is not noticeable in Fig. 2. Good results can be obtained from Eq. (B) with c = 0.365 when $H/D \leq 0.8$. The value of c should be increased about 4 per cent when H/D = 0.9 and 9 per cent when H/D = 1.0. A similar tendency for the coefficient of Eq. (B) to become nonuniform at high values of H/D has been noted previously by the writer in connection with the hood inlet.(2)

The average value of c in Eq. (B) is 0.365 whereas Mr. Stevens gives an average value of 0.59. The difference between the value of c obtained through the use of Eq. (B) and that obtained by Mr. Stevens through the use of Eq. (3) to compute the theoretical discharge may be ascribed to what Mavis(3) calls the "shape factor". As Professor Mavis points out, the shape factor for

1. Project Supervisor, U. S. Dept. of Agriculture, Agricultural Research Service, Soil and Water Conservation Research Div., St. Anthony Falls Hydr. Lab., Minneapolis, Minn.

circular weirs is variable. There is no shape factor as used by Mavis in Eq. (B) whereas there is a shape factor in the theoretically correct Eq. (3). The ratio of the discharge coefficient c determined through the use of Eq. (B) and the discharge coefficient c_s computed by Mr. Stevens is a measure of that portion of the shape factor not contained in the ratio a/A. The writer has computed c/c_s and plotted the values in Fig. B. The points should fall on a single line. Large deviations were found for some points but they were reduced when checks showed typographical or computational errors in the source data. No check was applied to the smaller deviations and it is likely that the data would define a better curve if such a check had been carried out. In any case, the curve of Fig. B is well defined.

No claim is made that the writer's approach is superior to the more rigorous method used by Mr. Stevens. It is simply another way of achieving the same end that the writer followed through because of his interest in it and his desire to make comparisons with Mr. Stevens' elliptic formula. The results are presented here in case others may have a similar interest.

Errata

No systematic check has been made of this paper but such errors as have been noted are listed.

In Eq. (1) the line immediately following, and the notation the letter ℓ is intended instead of the number 1.

Under "Application of the Elliptic Formula", second paragraph, line 2, D/H should read H/D.

The author of Reference 13 is Jorissen. In the title the words Circulaire and en should be separated.

In Appendix A:

H	H/D	Q_t	c
	E. R. Dodge, D = 0.211		
.193		.077	.649
.202		.083	.614
.211		.087	.632
Ave.			.626
	E. R. Dodge, D = 0.336		
.142		.065	.615
.161		.082	.610
Ave.			.600
	F. W. Greve, D = 0.75		
Ave.			.574
	H. V. Cone, D = 1.50		
Ave.			.579
	F. W. Greve, D = 2.25		
.525	.233		
Ave.			.598

H	H/D	Q_t	c
	F. W. Greve, D = 2.495		
.723			.586
Ave.			.595
	H. V. Cone, D = 3.00		
Ave.			.465

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2. Blaisdell, Fred W. and Donnelly, Charles A., Hydraulics of Closed Conduit Spillways, Part X. The Hood Inlet, University of Minnesota, St. Anthony Falls Hydraulic Laboratory Technical Paper No. 20, Series B, 36 pp., publication pending.
3. Mavis, F. T., "There's No Mystery in Weir-Flow Calculations", Eng. News-Rec., January 6, 1949, pp. 76-78.

W. T. MOODY.[1]—All too frequently when an engineer is confronted with an elliptic integral, such as is given by the author's Eq. (2), he is inclined to either abandon the analytical approach entirely, attempt to avoid the difficulty by making some simplifying assumption, or, at best, obtain an approximate solution in series form or by numerical integration. The author is to be commended on carrying out the onerous task of obtaining the closed expression for theoretical flow over a circular weir given by his Eq. (3). In addition his summary of available experimental data and his determination of a discharge coefficient is a valuable compilation for those who may wish to use this convenient method of measuring flow.

Until recently the only method of resolving problems of this type (involving elliptic integrals) was by making, for each individual case, such a lengthy development as is given in the author's Appendix D. However with the recent publication of Byrd and Friedman's excellent handbook(1) such tasks have been greatly simplified. An acquaintance with this reference book is well worth the time of engineers who may encounter problems of this nature.

It is doubly unfortunate that this handbook was apparently unavailable to the author. In addition to the saving of time and effort in the mathematical development, use of the six place tables contained in this reference might have saved many errors in the table contained in his Appendix C. These apparently resulted from using elliptic integral tables of insufficient accuracy. As a result, of the 101 entries in the first column of values of Δ, only 16 are correct in the fourth decimal place. None of these errors is in excess of 5 units in the fourth decimal place and in general they are therefore unimportant from a practical standpoint. However the writer would consider a three place table, known to be correct, preferable to a four place table of doubtful accuracy.

1. Engr., U. S. Dept. of the Interior, Bureau of Reclamation, Denver, Colo.

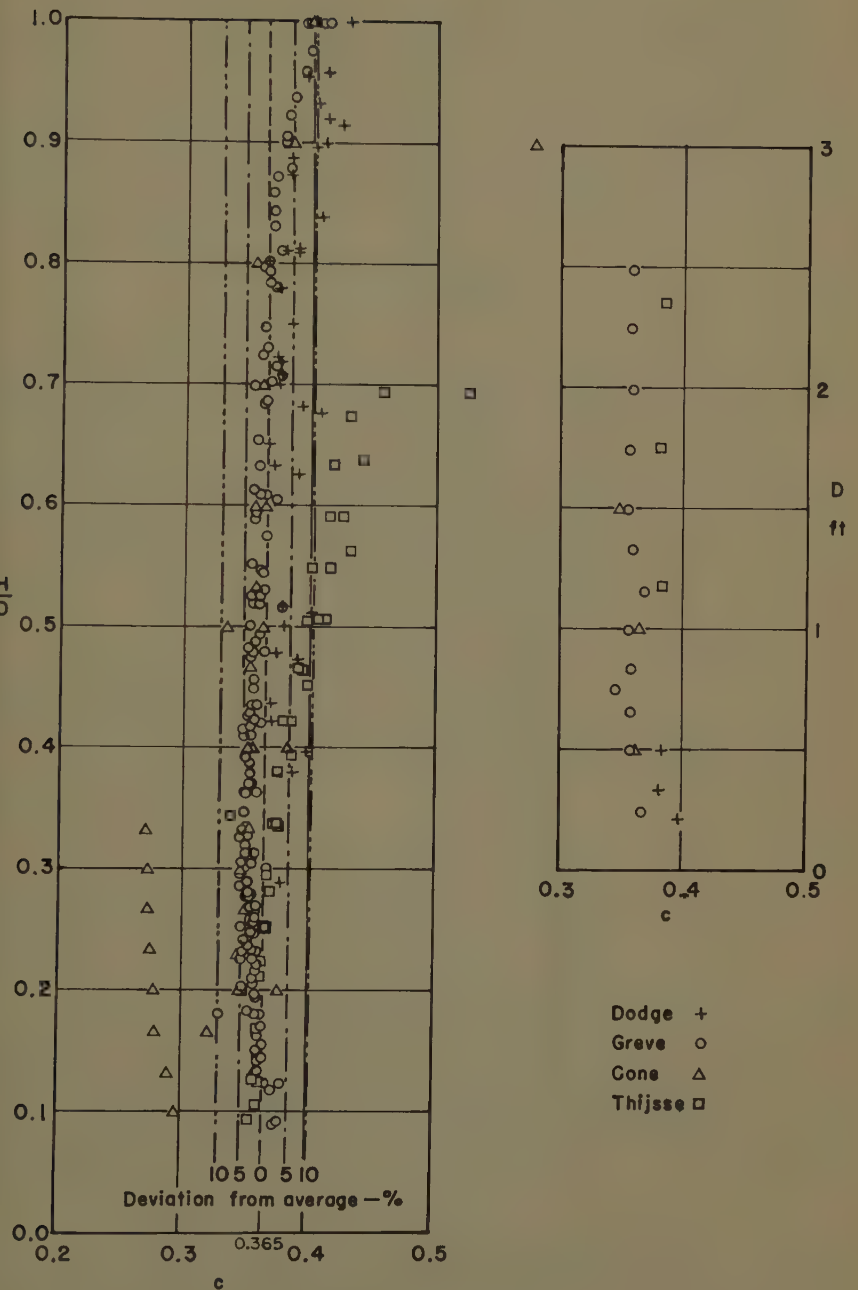
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
H/D
10 5 0 5 10
Deviation from average — %
0.2
0.3
0.365
0.4
0.5
c
3
2
1
0
D ft
0.3
0.4
0.5
c
Dodge +
Greve o
Cone △
Thijsse □

Fig. A

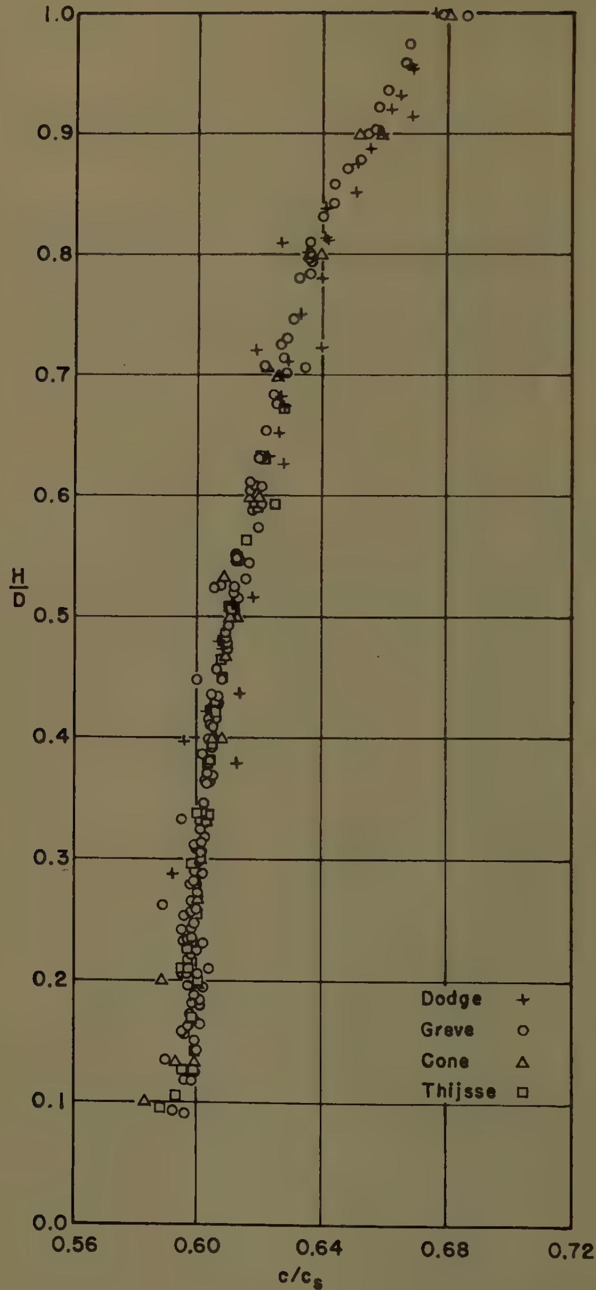
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
$\frac{H}{D}$
0.56
0.60
0.64
0.68
0.72
c/c_s
Dodge +
Greve ○
Cone △
Thijsse □

Fig. B

Table 1 gives values of the author's function Δ, correct to four decimal places for $k^2 = 0(0.01)1$, together with its second differences, δ^2. Values in this table were computed to six decimal places,[2] checked by differencing, and rounded to four decimal places. The abbreviated form of the table seems justified since the second differences are everywhere small. Because these differences are nearly constant the maximum error of linear interpolation will never be greater than 1/8 of the second difference and therefore usually less than the possible rounding error of the table. Where greater precision is required, second degree interpolation using either Everett's or Lagrange's formula will give results to the full accuracy of the table.

It can be shown that

$$\operatorname*{Limit}_{k \to o} \Delta = \frac{15\,\pi\,k^4}{16} \tag{1}$$

For small values of k, when there is loss of significant figures in the table, this approximation may be used with good results. The relative error of this approximation is very nearly $+k^2/4$ so that it is possible to determine Δ at any point within an accuracy of one per cent.

The function Δ may also be used to compute theoretical flow through a circular orifice. Its use should be particularly appropriate where such flow takes place under low heads.

With H still defined as the head above the invert (this time of the orifice) the author's differential relation still holds. Now however

$$Q_t = 2\sqrt{2g}\int_o^D \sqrt{Y\,(H-Y)(D-Y)}\;dy \tag{2}$$

Using formulas 233.08 and 361.04 of the reference in footnote 2 with, in this case, $k^2 = D/H$, the result is obtained directly as

$$Q_t = \frac{4}{15}\,D^2\sqrt{2gH}\,\frac{1}{k^4}\left[\,2\,(1-k^2+k^4)E-(2-k^2)\,(1-k^2)\,K\,\right] \tag{3}$$

This can be written as

$$Q_t = \frac{16\Delta}{15\pi k^4}\;A\sqrt{2gH}, \tag{4}$$

where A represents the area of the orifice. From the relation (1) it is seen that for $D << H$ the flow takes the familiar form

$$Q_t = A\sqrt{2gH}. \tag{5}$$

Here, however, H is measured to the invert of the orifice as noted above instead of as is usually done, to the center of the opening. This fact suggests that better correlation of coefficients for orifice flow, particularly under low heads, might be obtained if they were applied to the theoretical results (3) or (4) instead of the usual equation expressing the flow in terms of the head on the center of the opening.

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2. For a less expensive and perhaps more readily available table of elliptic integrals see: Milne-Thomson, L. M.; Jacobian Elliptic Function Tables; Dover, New York, 1950.

M. JEAN RIGARD.[1]—It is felt that a few comments on the additional research carried out by the late Mr. Jorissen on this subject after the work described in his article in "La Revue Universelle des Mines" (8e série, Tome XIV—12.12.38—Author's reference no. 13) would not be out of place.

Remarkable powers of synthesis are revealed by his article entitled "A contribution to research on sharp-crested circular weirs" (Revue Générale de l'Hydraulique, Jan/Feb. 1943, p. 9) and his very thorough investigation, based on dimensional analysis methods, of the results of tests carried out on the subject by a number of authors (Hegly, Staus, Sanden, Ramponi, Marchetti, and finally Ferroglio). He reaches the interesting conclusion that, all other things being equal, all these experiments show the same discharge coefficient variations to within approximately ± 1%, if the velocity of approach (which is governed by the characteristics of the system) is allowed for, and that, although there are some special cases—such as his own initial test series and Ferroglio's experiments—where this is not true, these are very few and occur for perfectly logical reasons which he analyses quite convincingly.

The similarity between these various test results can however only hold good if the following conditions are satisfied:

a. The weir must be sharp-crested, with a smooth vertical surface.
b. The supply channel must be smooth-walled, horizontal, with a rectangular cross-section, and be long enough to provide a uniform velocity distribution in the section in which the head is measured.
c. The vein of water must not be affected by the downstream level.
d. Head readings must be taken sufficiently far upstream of the weir to be able to neglect drawdown effects.
e. The weir diameter must be less than 500 mm.
f. The ratio between the width of the supply channel and the height of the weir centre above the canal invert ($\frac{\alpha}{\beta}$ = "form factor" for the weir) must be less than, or at most, equal to 2.
   (Although the late Mr. A. Jorissen and his team had intended to carry out further measurements at Liège University in connection with condition (f) above, which is somewhat restrictive, the results of these tests have not been published as far as we can ascertain).

If the above conditions are satisfied, coefficient C in Staus and Sanden's general formula:

$$Q = C.\ qi.\ d^2 \sqrt{d}$$

can be expressed in the form recommended by Mr. A. Jorissen for water, as follows:

$$C = \left(0{,}664\ d^{-0{,}025} + 0{,}085 - \frac{\frac{\omega}{d^2}}{10\ m}\right) \left[1 + \left(\frac{\omega}{\Omega}\right)^2\right]$$

1. Engr., "Société Grenobloise d=Etudes et d'Applications Hydrauliques" (SOGREAH), Grenoble, France.

Table 1

Values of

$$\Delta = 2(1 - k^2 + k^4)E - (2 - k^2)(1 - k^2)K$$

k^2	Δ	δ^2	k^2	Δ	δ^2	k^2	Δ	δ^2	k^2	Δ	δ^2
0	0	6									
0.01	0.0003	6	0.26	0.1856	5	0.51	0.6587	3	0.76	1.3227	1
.02	.0012	6	.27	.1995	5	.52	.6824	3	.77	.3515	1
.03	.0026	6	.28	.2140	5	.53	.7063	3	.78	.3803	1
.04	.0047	6	.29	.2288	4	.54	.7306	3	.79	.4092	1
.05	.0073	6	.30	.2441	4	.55	.7551	3	.80	.4381	0
0.06	0.0104	6	0.31	0.2599	4	0.56	0.7799	3	0.81	1.4671	0
.07	.0142	6	.32	.2761	4	.57	.8050	3	.82	.4961	0
.08	.0185	6	.33	.2927	4	.58	.8304	3	.83	.5251	0
.09	.0233	5	.34	.3098	4	.59	.8560	3	.84	.5541	0
.10	.0287	5	.35	.3272	4	.60	.8819	2	.85	.5831	0
0.11	0.0346	5	0.36	0.3451	4	0.61	0.9080	2	0.86	1.6121	-1
.12	.0411	5	.37	.3634	4	.62	.9343	2	.87	.6410	-1
.13	.0481	5	.38	.3821	4	.63	.9609	2	.88	.6698	-1
.14	.0557	5	.39	.4012	4	.64	.9877	2	.89	.6985	-1
.15	.0637	5	.40	.4207	4	.65	1.0147	2	.90	.7271	-1
0.16	0.0723	5	0.41	0.4405	4	0.66	1.0418	2	0.91	1.7556	-2
.17	.0814	5	.42	.4607	4	.67	.0692	2	.92	.7839	-2
.18	.0910	5	.43	.4814	4	.68	.0968	2	.93	.8120	-2
.19	.1011	5	.44	.5023	4	.69	.1245	2	.94	.8399	-3
.20	.1117	5	.45	.5236	4	.70	.1524	2	.95	.8675	-3
0.21	0.1228	5	0.46	0.5453	3	0.71	1.1805	1	0.96	1.8949	-3
.22	.1344	5	.47	.5673	3	.72	.2087	1	.97	.9219	-4
.23	.1465	5	.48	.5897	3	.73	.2370	1	.98	.9484	-5
.24	.1590	5	.49	.6124	3	.74	.2655	1	.99	.9745	-6
.25	.1721	5	.50	.6354	3	.75	.2941	1	1.00	2.0000	-9

where:

d = weir diameter

ω = wetted section of the weir beneath the upstream water level, which is equivalent to a height of segment h (equal to the head)

$m = \frac{h}{d}$, the ratio between the head on the weir and the diameter of the latter

Ω = wetted section of the supply canal

qi being given by the following relation established by Staus and Sanden:

$$qi = \frac{4}{15}\sqrt{2g}\left[2\left(1 - \frac{h}{d} + \frac{h^2}{d^2}\right)E - \left(2 - \frac{3h}{d} + \frac{h^2}{d^2}\right)K\right]$$

(where E and K are two elliptic integrals of the 1st and 2nd kind, the value of which solely depends on $\frac{h}{d}$).

Despite its apparent complexity, coefficient C can be quickly found as a function of $m = \frac{h}{d}$ in most cases, with the aid of diagrams and tables supplied by Mr. A. Jorissen, from which the discharge can finally be calculated to within ± 1%.

M. B. McPHERSON,[1] A. M. ASCE.—The final steps were omitted in the original derivation, by Sanden in reference (9), of the author's Eq. (3)—(this reference was authored by both A. Staus and K. von Sanden, the latter being credited for the theory. Omori[2] published a complete derivation some time ago, using a different approach from either the author's or the original, with an intermediate function $Y = HZ^2$.

Appendices A and C, but particularly the latter, are certainly worthwhile contributions. The data from reference (9) was unfortunately omitted from Appendix A. The author recognizes need for evaluating the effect of approach conditions but does not provide any approach dimensions. It is hoped that this information will be included in the closure to discussion, particularly for the Dodge tests.

The author states: "The Greve experiments are the most consistent of all and are believed to be entirely trustworthy." (The Greve bulletin was published in March, 1932). The writer has tabulated in Table A values of C obtained by attempting to get a best fit to the Greve data; 90% of the experimental data have values which are within 2% of the tabulated. Omori presented flow rates representing a best fit to his data, which have been converted to values of C; the test data would be within 1% of the tabulation for H/D greater than 0.3 and within 2.2% for H/D less than 0.3. Staus(12) presented an empirical equation for C which was representative of his data within about 2%; values from his equation are given in Table A. Staus and Sanden(9) listed all test data, but also gave (in their Table 3) trend or best fit values of C which were also within about 2%; these are tabulated in Table A.

1. Research Engr., Philadelphia Water Dept., Philadelphia, Pa.
2. Omori, Tokusaku, "The Discharge over Circular Weirs (in Japanese), Journal Society of Mech. Engrs., Japan, Vol. 36, No. 197, pp. 598-601, September, 1933.

Smoothing the numerical average of the above four tabulations (last column, ble A) yields an agreement with the author's proposed C of 0.59, as long as D is greater than about 0.25.

It is interesting to note that the four groups of experiments represent newhat different approach conditions:

				Test Tank (Approach)		
By	Ref.	No. of Weirs Tested	Range of Diam.	Width, B	Floor to ℄ Wier	Max. D/B
ori	(Footnote c)	5	150-250MM	1.39M	0.67M	0.18
us/Sanden	(9)	7	150-300MM	0.60M	0.50M	0.50
us	(12)	5	200-300MM	2D	1.5 D	0.50
eve	(2)	13	0.25-2.50 ft	5 ft.	?	0.50

e Omori tests were for a maximum D/B of 0.18 whereas the remainder re for a much narrower approach. In Table A, for H/D greater than about , the Omori and Greve values are quite similar; likewise the Staus and the us and Sanden values. The Omori data should reflect the least approach ect. Were it not for the Greve data, the Omori tests might well be con- ered as the limit of negligible approach effects.

The Omori data are closely satisfied with C = 0.58 for H/D greater than out 0.4. In Table A, the variations in C for H/D less than about 0.3 are no ıbt due to surface tension and/or viscous effects, but the scatter of the ginal data precludes an approximate comparative appraisal of these effects m one diameter range to the next.

Omori was possibly the only investigator to carry his experiments to an D as high as 1.20. The head on an orifice = H-D/2 = h. In Table B are ulated equivalent discharge coefficients for an orifice using the square- t of h and flows given by Omori (C_{or}). Also shown are flow rates for a -foot diameter orifice assuming C_{or} = 0.62, for comparison with the rates a one-foot weir using typical characteristics by Omori. A comparative t of these rates indicates that orifice-type flow would occur at an H/D newhere between 1.5 and 2.0 (or h/D from 1.0 to 1.5). The safe lower limit orifice flow is often considered to be h/D = 2 in various handbooks.

STEPONAS KOLUPAILA.[1]—The writer appreciates the admirable intention he author to popularize the simple, convenient, and accurate type of circu- weir. The only obstacle to promote this device into practice was its mula containing two elliptical integrals. The author carefully repeated the k which was accomplished in Europe some 25 years ago and which, un- tunately, remains very little known here.

The writer cannot agree with statement that "most of the European atises on such weirs are not generally available now": every good library leading foreign magazines on its shelves. What we really need is a tematic bibliographical index of literature which could save many efforts much time wasted in searches and duplication of work already done.

The writer would like to add some historical remarks:

The oldest notice on circular weirs was an abstract of a report read before Royal Society of Edinborough in 1908 by G. H. Gulliver, who presented a

Prof. of Civ. Eng., Univ. of Notre Dame, Notre Dame, Ind.

TABLE A
(Disc: 1455)

Comparison of Experimental Results -
Tests on Circular Weirs

Coefficient of Discharge - C

H/D	Omori (Ref. - Footnote c)	Staus (Ref. 12) (From Equat.)	Staus & Sanden (Ref. 9)	Greve (Ref.2) (Best Fit)	Average For All Four
0.10	.666	.650	.646	.624	.646
0.15	.620	.622	.629	--	--
0.20	.619	.609	.617	.592	.609
0.25	.611	.602	--	--	--
0.30	.601	.598	.604	.581	.596
0.35	.594	.595	--	--	--
0.40	.589	.594	.597	.577	.589
0.45	.586	.594	--	--	--
0.50	.584	.594	.593	.576	.587
0.55	.581	.594	--	--	--
0.60	.580	.595	.590	.576	.585
0.65	.579	.596	--	--	--
0.70	.578	.597	.590	.575	.585
0.75	.578	.598	--	--	--
0.80	.578	.599	.590	.574	.585
0.85	.578	.601	--	--	--
0.90	.579	.602	.594	.575	.587
0.95	.581	.604	--	--	--
1.00	.584	.605	.598	.582	.592

TABLE B

(Disc. 1455)

Weir vs. Orifice Flow

(Flow Rates in Terms of a One-Foot Dia. Weir)

Weir, H/D	Q*(Omori)	2.14 △	C	Orifice, h/D	Equiv. $C_{or.}$	Q for $C_{or.}$ = 0.62
1.00	2.50 cfs	4.28 cfs	.584	.50	.560	2.76 cfs
1.05	2.66	4.60	.578	.55	.568	2.90
1.10	2.80	4.88	.575	.60	.573	3.03
1.15	2.93	5.18	.565	.65	.577	3.15
1.20	3.05	5.45	.560	.70	.579	3.27
1.50	--	--	--	1.00	.620?	3.91
2.00	--	--	--	1.50	.620	4.79

(*From best-fit characteristics).

semi-graphical method for calculating of water discharge through the upper part of the notch.

A series of weir investigations were performed in the Experimental Stati of the U. S. Dept. of Agriculture at Fort Collins, Colo., in 1915, by V. M. Co (Victor Mann Cone, inventor of the Venturi flume, not H. V., as mistakenly printed in the Paper 1455). Cone derived exact formulas for triangular notcl and presented graphically the results of testing the circular weir. M. P. O'Brien in 1928, then assistant at the Purdue University, computed an empir cal equation from Cone's data, as

$$Q = 2.747\ h^{1.807}\ d^{0.693},$$

equivalent to

$$Q = 2.747\ x^{1.807}\ d^{2.5}$$

where

$$x = h/d.$$

The next to investigate the circular weir and publish a paper, was a Finni engineer John L. W. Lillja. In 1920 he skillfully derived a very accurate formula

$$q = -0.000145\ x + 0.791\ x^2 - 0.226\ x^3$$

for a general equation

$$Q = c_d\ q\ d^{\,2.5}.$$

A series of good tests were performed in 1921-1928 at Purdue University Lafayette, Ind., under direction of Prof. F. W. Greve. In 1932 Greve publish results of these tests and added an excellent bibliographical review of 53 papers in English language. Greve obtained an empirical equation for circul weirs

$$Q = 2.87\ h^{1.87}\ d^{0.637},$$

equivalent to

$$Q = 2.87\ x^{1.87}\ d^{2.5}$$

The true promotor of the circular weir was Dr. Eng. Anton Staus, professor of the Technical College at Esslingen, Germany, since 1922. In coop ation with K. von Sanden, professor of mathematics in the Karlsruhe Politechnical Institute, he derived a theory of the circular weir and computed a table of values, involving elliptical integrals. This table, published in 1930, contains values of derivation with 6 decimals for every 0.01 x. The exact formula developed by Staus was

$$q = \frac{4}{15}\sqrt{2g}\left[\,2\ (1 - x + x^2)\ E - (2 - 3x + x^2)\ K\,\right]$$

where E and K are complete elliptical integrals of two kinds. The tabular values are to be multiplied by $\frac{4}{15}\sqrt{2g} = 1.182$ for metric units or 2.14 for English units. A table of q in English units for every 0.01 of $x = h/d$ follows

x	q	x	q	x	q	x	q
0.00	0.00000	0.25	0.368	0.50	1.360	0.75	2.769
0.01	0.00063	0.26	0.397	0.51	1.410	0.76	2.830
0.02	0.00252	0.27	0.427	0.52	1.461	0.77	2.892
0.03	0.00562	0.28	0.458	0.53	1.512	0.78	2.954
0.04	0.00998	0.29	0.490	0.54	1.564	0.79	3.016
0.05	0.01555	0.30	0.522	0.55	1.616	0.80	3.078
0.06	0.0223	0.31	0.556	0.56	1.669	0.81	3.140
0.07	0.0303	0.32	0.590	0.57	1.723	0.82	3.202
0.08	0.0395	0.33	0.626	0.58	1.777	0.83	3.264
0.09	0.0499	0.34	0.662	0.59	1.831	0.84	3.326
0.10	0.0614	0.35	0.700	0.60	1.887	0.85	3.388
0.11	0.0741	0.36	0.738	0.61	1.943	0.86	3.450
0.12	0.0880	0.37	0.777	0.62	1.999	0.87	3.512
0.13	0.1030	0.38	0.817	0.63	2.055	0.88	3.573
0.14	0.1191	0.39	0.858	0.64	2.112	0.89	3.635
0.15	0.1363	0.40	0.900	0.65	2.170	0.90	3.696
0.16	0.1547	0.41	0.942	0.66	2.229	0.91	3.757
0.17	0.1741	0.42	0.986	0.67	2.288	0.92	3.817
0.18	0.1948	0.43	1.030	0.68	2.346	0.93	3.878
0.19	0.2162	0.44	1.075	0.69	2.405	0.94	3.937
0.20	0.2390	0.45	1.120	0.70	2.465	0.95	3.996
0.21	0.2628	0.46	1.166	0.71	2.525	0.96	4.055
0.22	0.2875	0.47	1.214	0.72	2.586	0.97	4.113
0.23	0.3135	0.48	1.262	0.73	2.647	0.98	4.170
0.24	0.3401	0.49	1.311	0.74	2.708	0.99	4.226
0.25	0.3680	0.50	1.360	0.75	2.769	1.00	4.280

Staus investigated 5 weirs of diameters 0.65 to 1.00 ft. Tests proved that e discharge coefficient c_d is not constant, but varies with x. Staus designed empirical formula

$$c_d = 0.555 + \frac{1}{110\,x} + 0.041\,x.$$

S. Gradstein and A. Walther (1931) offered a method of graphical determi-tion of q values.

F. Ramponi (1936) proposed an approximate formula, correct to 0.5%, for lues of q in the Staus formula

$$q = 10.12\,x^{1.975} - 2.66\,x^{3.78}.$$

A. Jorissen continued the work of Staus in the Hydraulic laboratory of the iversity of Liège. He investigated the influence of the size of channel of proach on the discharge coefficient; he suggested (1938) a correction to the us formula for the width b of the channel:

$$\left(\frac{2}{b/d}\right)^{0.0625}$$

while influence of height of the notch over the bottom was found negligible when the center of the notch is more than 1.2 d from the bottom. No corrections are necessary when the channel is wider than 2d.

Mr. Stevens has correctly derived the same formula, as did Staus. With the assistance of Prof. W. E. Milne he prepared a table of values for every 0.001; data are given with 4 decimals, and the fourth decimal differs from data in the Staus table with 6 decimals. Certainly, this is not important for practical use. From analysis of 23 series of tests by different experimenters the author selected the average value of the discharge coefficient 0.59 and prepared a table for circular weirs 0.25 to 5.00 ft. in diameter.

The writer would like to add some results of calibration of circular weirs in the Hydraulics laboratory of the University of Notre Dame. A sharp-edged circular notch, 1.00 ft. in diameter, is located in a flume 2.25 ft. wide, its center is 1.75 ft. above the bottom. Volumetric measurements show a systematic difference about 2% from the empirical formula by Staus. Closer agreement is obtained with a corrected formula

$$c_d = 0.570 + \frac{1}{100x} + 0.041\,x.$$

The fact that the discharge coefficient c_d varies with relative height suggests a conclusion that derivation of the equation for the circular weir is somewhat inaccurate. The error lies probably in the curvature of stream lines before the weir or in different heights before the weir and over its crest.

It is easy to derive an entirely empirical equation by plotting Q and h on logarithmic paper and tracing a straight line, adequate in limits 0.075 x to 0.75 x. A formula for a 1.00-ft. weir, accurate to ± 2%, is

$$Q = 2.85\,h^{1.85}.$$

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TURBULENCE CHARACTERISTICS OF THE HYDRAULIC JUMP[a]

Discussion by Edward Silberman

EDWARD SILBERMAN,[1] A. M. ASCE.—It is surprising that the authors, in a paper dealing so intimately with the fundamental mechanics of flow in the hydraulic jump, should have failed to call attention to an important phenomenon associated with the jump. The phenomenon referred to is the abrupt separation of the boundary layer under the beginning of the jump produced by the rapid decrease in velocity. This separation is identical with that occurring at transition from a parallel-sided to a diverging flume, for example, and is accompanied by a similar eddying motion under the jump. There are, in fact, two rollers at the jump—one on the surface and one on the floor—and similar turbulence phenomena are associated with both.

The writer uses a simple experiment to demonstrate separation under the jump to his classes. A 6-in. wide, glass-sided, smooth-bottom flume is prepared by connecting a dye tube to the outside of a piezometer tap located on the bottom of the channel. A jump is established in the channel so as to move downstream, passing over the piezometer tap. Observation of a dye stream emitted very slowly from the tap as the jump passes yields the following information:

a. In the subcritical region, dye leaves the hole in a downstream direction, as would be expected in a normal boundary layer.
b. As the end of the upper roller reaches the hole, the dye stream begins to waver. The general motion is still downstream, but wisps of dye are carried along the bottom to both sides.
c. Further within the roller, the dye has no characteristic direction, indicating that there is no boundary layer at all; complete separation has occurred.
d. From about a half to a third of the length of the roller forward to very near the beginning of the jump, the dye stream on the bottom actually moves upstream, indicating complete reversal of the boundary layer and the existence of a bottom roller.
e. Just downstream of the beginning of the jump there is an abrupt change in direction of the dye from upstream to downstream again. A stagnation point and the beginning of the roller occur at this point.

The phenomenon described can be observed at Froude numbers at least as small as 3, and for all higher numbers.

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a. Proc. Paper 1528, February, 1958, by Hunter Rouse, T. T. Siao, and S. Nagaratnam.

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The authors did not publish vertical velocity traverses through the jump; these should have given some indication of the existence of the bottom roller. It is noted, however, that the jump streamlines shown in Fig. 5 of the paper are displaced upward with respect to the air-flow streamlines; this displacement is probably a consequence of the phenomenon under discussion. Separation on the bottom under the jump had already been referred to by M. P. O'Brien, M., ASCE, in his discussion of the Bakhmeteff and Matzke paper.(1)

In view of the existence of this bottom roller, there is some question as to whether the turbulence characteristics of the jump have been adequately modelled by the subsonic air-flow model used by the authors. It is suggested that the use of a sonic throat at entrance to the model, accompanied by a normal shock wave in the diverging portion of the duct would probably be a better model of the jump. Similar separation on a straight wall occurs behind a shock wave as occurs behind the hydraulic jump.

PROCEEDINGS PAPERS

The technical papers published in the past year are identified by number below. Technical-division nsorship is indicated by an abbreviation at the end of each Paper Number, the symbols referring to: Air insport (AT), City Planning (CP), Construction (CO), Engineering Mechanics (EM), Highway (HW), Hyulics (HY), Irrigation and Drainage (IR), Pipeline (PL), Power (PO), Sanitary Engineering (SA), Soil chanics and Foundations (SM), Structural (ST), Surveying and Mapping (SU), and Waterways and Harbors V), divisions. Papers sponsored by the Board of Direction are identified by the symbols (BD). For es and order coupons, refer to the appropriate issue of "Civil Engineering." Beginning with Volume 82 nuary 1956) papers were published in Journals of the various Technical Divisions. To locate papers in Journals, the symbols after the paper numbers are followed by a numeral designating the issue of a parılar Journal in which the paper appeared. For example, Paper 1449 is identified as 1449 (HY 6) which icates that the paper is contained in the sixth issue of the Journal of the Hydraulics Division during 1957.

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NE: 1260(HY3), 1261(HY3), 1262(HY3), 1263(HY3), 1264(HY3), 1265(HY3), 1266(HY3), 1267(PO3), 1268 (PO3), 1269(SA3), 1270(SA3), 1271(SA3), 1272(SA3), 1273(SA3), 1274(SA3), 1275(SA3), 1276(SA3), 1277 (HY3), 1278(HY3), 1279(PL2), 1280(PL2), 1281(PL2), 1282(SA3), 1283(HY3)^c, 1284(PO3), 1285(PO3), 1286 (PO3), 1287(PO3)^c, 1288(SA3)^c.

LY: 1289(SM3), 1290(EM3), 1291(EM3), 1292(EM3), 1293(EM3), 1294(HW3), 1295(HW3), 1296(HW3), 1297 (HW3), 1298(HW3), 1299(SM3), 1300(SM3), 1301(SM3), 1302(ST4), 1303(ST4), 1304(ST4), 1305(SU1), 1306 (SU1), 1307(SU1), 1308(ST4), 1309(SM3), 1310(SU1)^c, 1311(EM3)^c, 1312(ST4), 1313(ST4), 1314(ST4), 1315 (ST4), 1316(ST4), 1317(ST4), 1318(ST4), 1319(SM3)^c, 1320(ST4), 1321(ST4), 1322(EM3), 1323(AT1), 1324 (AT1), 1325(AT1), 1326(AT1), 1327(AT1), 1328(AT1)^c, 1329(ST4)^c.

GUST: 1330(HY4), 1331(HY4), 1332(HY4), 1333(SA4), 1334(SA4), 1335(SA4), 1336(SA4), 1337(SA4), 1338 (SA4), 1339(CO1), 1340(CO1), 1341(CO1), 1342(CO1), 1343(CO1), 1344(PO4), 1345(HY4), 1346(PO4)^c, 1347 (BD1), 1348(HY4)^c, 1349(SA4)^c 1350(PO4), 1351(PO4).

PTEMBER: 1352(IR2), 1353(ST5), 1354(ST5), 1355(ST5), 1356(ST5), 1357(ST5), 1358(ST5), 1359(IR2), 1360 (IR2), 1361(ST5), 1362(IR2), 1363(IR2), 1364(IR2), 1365(WW3), 1366(WW3), 1367(WW3), 1368(WW3), 1369 (WW3), 1370(WW3), 1371(HW4), 1372(HW4), 1373(HW4), 1374(HW4), 1375(PL3), 1376(PL3), 1377(IR2)^c, 1378 (HW4)^c, 1379(IR2), 1380(HW4), 1381(WW3)^c, 1382(ST5)^c, 1383(PL3)^c, 1384(IR2), 1385(HW4), 1386(HW4).

TOBER: 1387(CP2), 1388(CP2), 1389(EM4), 1390(EM4), 1391(HY5), 1392(HY5), 1393(HY5), 1394(HY5), 1395 (HY5), 1396(PO5), 1397(PO5), 1398(PO5), 1399(EM4), 1400(SA5), 1401(HY5), 1402(HY5), 1403(HY5), 1404 (HY5), 1405(HY5), 1406(HY5), 1407(SA5), 1408(SA5), 1409(SA5), 1410(SA5), 1411(SA5), 1412(EM4), 1413 (EM4), 1414(PO5), 1415(EM4)^c, 1416(PO5)^c, 1417(HY5)^c, 1418(EM4), 1419(PO5), 1420(PO5), 1421(PO5), 1422(SA5)^c, 1423(SA5), 1424(EM4), 1425(CP2).

VEMBER: 1426(SM4), 1427(SM4), 1428(SM4), 1429(SM4), 1430(SM4)^c, 1431(ST6), 1432(ST6), 1433(ST6), 1434(ST6), 1435(ST6), 1436(ST6), 1437(ST6), 1438(SM4), 1439(SM4), 1440(ST6), 1441(ST6), 1442(ST6)^c, 1443(SU2), 1444(SU2), 1445(SU2), 1446(SU2), 1447(SU2), 1448(SU2)^c.

CEMBER: 1449(HY6), 1450(HY6), 1451(HY6), 1452(HY6), 1453(HY6), 1454(HY6), 1455(HY6), 1456(HY6)^c, 1457(PO6), 1458(PO6), 1459(PO6), 1460(PO6)^c, 1461(SA6), 1462(SA6), 1463(SA6), 1464(SA6), 1465(SA6), 1466(SA6)^c, 1467(AT2), 1468(AT2), 1469(AT2), 1470(AT2), 1471(AT2), 1472(AT2), 1473(AT2), 1474(AT2), 1475(AT2), 1476(AT2), 1477(AT2), 1478(AT2), 1479(AT2), 1480(AT2), 1481(AT2), 1482(AT2), 1483(AT2), 1484(AT2), 1485(AT2)^c, 1486(BD2), 1487(BD2), 1488(PO6), 1489(PO6), 1490(BD2), 1491(BD2), 1492(HY6), 1493(BD2).

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IUARY: 1494(EM1), 1495(EM1), 1496(EM1), 1497(IR1), 1498(IR1), 1499(IR1), 1500(IR1), 1501(IR1), 1502 (IR1), 1503(IR1), 1504(IR1), 1505(IR1), 1506(IR1), 1507(IR1), 1508(ST1), 1509(ST1), 1510(ST1), 1511(ST1), 1512(ST1), 1513(WW1), 1514(WW1), 1515(WW1), 1516(WW1), 1517(WW1), 1518(WW1), 1519(ST1), 1520 (EM1)^c, 1521(IR1)^c, 1522(ST1)^c, 1523(WW1)^c, 1524(HW1), 1525(HW1), 1526(HW1)^c, 1527(HW1).

BRUARY: 1528(HY1), 1529(PO1), 1530(HY1), 1531(HY1), 1532(HY1), 1533(SA1), 1534(SA1), 1535(SM1), 1536(SM1), 1537(SM1), 1538(PO1)^c, 1539(SA1), 1540(SA1), 1541(SA1), 1542(SA1), 1543(SA1), 1544(SM1), 1545(SM1), 1546(SM1), 1547(SM1), 1548(SM1), 1549(SM1), 1550(SM1), 1551(SM1), 1552(SM1), 1553(PO1), 1554(PO1), 1555(PO1), 1556(PO1), 1557(SA1)^c, 1558(HY1)^c, 1559(SM1)^c

RCH: 1560(ST2), 1561(ST2), 1562(ST2), 1563(ST2), 1564(ST2), 1565(ST2), 1566(ST2), 1567(ST2), 1568 (WW2), 1569(WW2), 1570(WW2), 1571(WW2), 1572(WW2), 1573(WW2), 1574(PL1), 1575(PL1), 1576(ST2)^c, 1577(PL1), 1578(PL1)^c, 1579(WW2)^c.

RIL: 1580(EM2), 1581(EM2), 1582(HY2), 1583(HY2), 1584(HY2), 1585(HY2), 1586(HY2), 1587(HY2), 1588 (HY2), 1589(IR2), 1590(IR2), 1591(IR2), 1592(SA2), 1593(SU1), 1594(SU1), 1595(SU1), 1596(EM2), 1597(PO2), 1598(PO2), 1599(PO2), 1600(PO2), 1601(PO2), 1602(PO2), 1603(HY2), 1604(EM2), 1605(SU1)^c, 1606(SA2), 1607(SA2), 1608(SA2), 1609(SA2), 1610(SA2), 1611(SA2), 1612(SA2), 1613(SA2), 1614(SA2)^c, 1615(IR2)^c, 1616 (HY2)^c, 1617(SU1), 1618(PO2)^c, 1619(EM2)^c, 1620(CP1).

Y: 1621(HW2), 1622(HW2), 1623(HW2), 1624(HW2), 1625(HW2), 1626(HW2), 1627(HW2), 1628(HW2), 1629 (ST3), 1630(ST3), 1631(ST3), 1632(ST3), 1633(ST3), 1634(ST3), 1635(ST3), 1636(ST3), 1637(ST3), 1638(ST3), 1639(WW3), 1640(WW3), 1641(WW3), 1642(WW3), 1643(WW3), 1644(WW3), 1645(SM2), 1646(SM2), 1647 (SM2), 1648(SM2), 1649(SM2), 1650(SM2), 1651(HW2), 1652(HW2)^c, 1653(WW3)^c, 1654(SM2), 1655(SM2), 1656(ST3)^c, 1657(SM2)^c.

IE: 1658(AT1), 1659(AT1), 1660(HY3), 1661(HY3), 1662(HY3), 1663(HY3), 1664(HY3), 1665(SA3), 1666 (PL2), 1667(PL2), 1668(PL2), 1669(AT1), 1670(PO3), 1671(PO3), 1672(PO3), 1673(PL2), 1674(PL2), 1675 (PO3), 1676(PO3), 1677(SA3), 1678(SA3), 1679(SA3), 1680(SA3), 1681(SA3), 1682(SA3), 1683(PO3), 1684 (HY3), 1685(SA3), 1686(SA3), 1687(PO3), 1688(SA3)^c, 1689(PO3)^c, 1690(HY3)^c, 1691(PL2)^c.

Discussion of several papers, grouped by divisions.

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